Dethroning the median voter

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Games and Optimization 2022
St. Etienne

13-15 April 2022

Acknowledgement

The author is grateful for the Bolyai Research Scholarship provided by the Hungarian Academy of Sciences and the New National Excellence Program Bolyai+ scholarship provided by the Ministry for Innovation and Technology.

Newspaper headlines from recent years

A precursor to the Median Voter Theorem

Hotelling (1929)

Suppose two vendors would like to open hot dog stands on the beach.

- Prospective customers spread over the beach uniformly.
- Customers choose the closest stand.

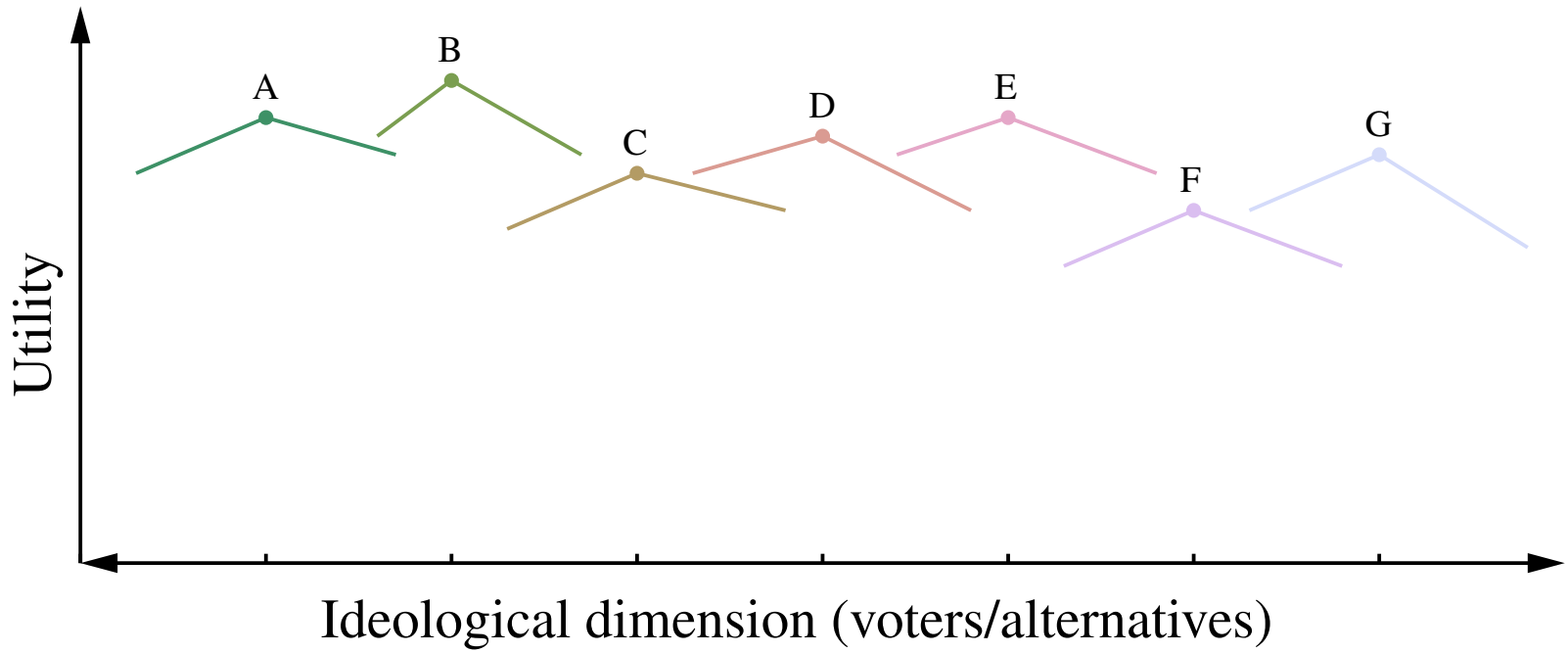
Then both stands will set up at the middle of the beach.

This observation has applications in a wide variety of fields, including political science and marketing. Note that if vendors engage in price competition then the result reverses: they choose the opposite side of the beach. (D'Aspremont, C.; Gabszewicz, J. Jaskold; Thisse, J.-F. (1979).)

Median voter theorem (weak version)

Black (1948)

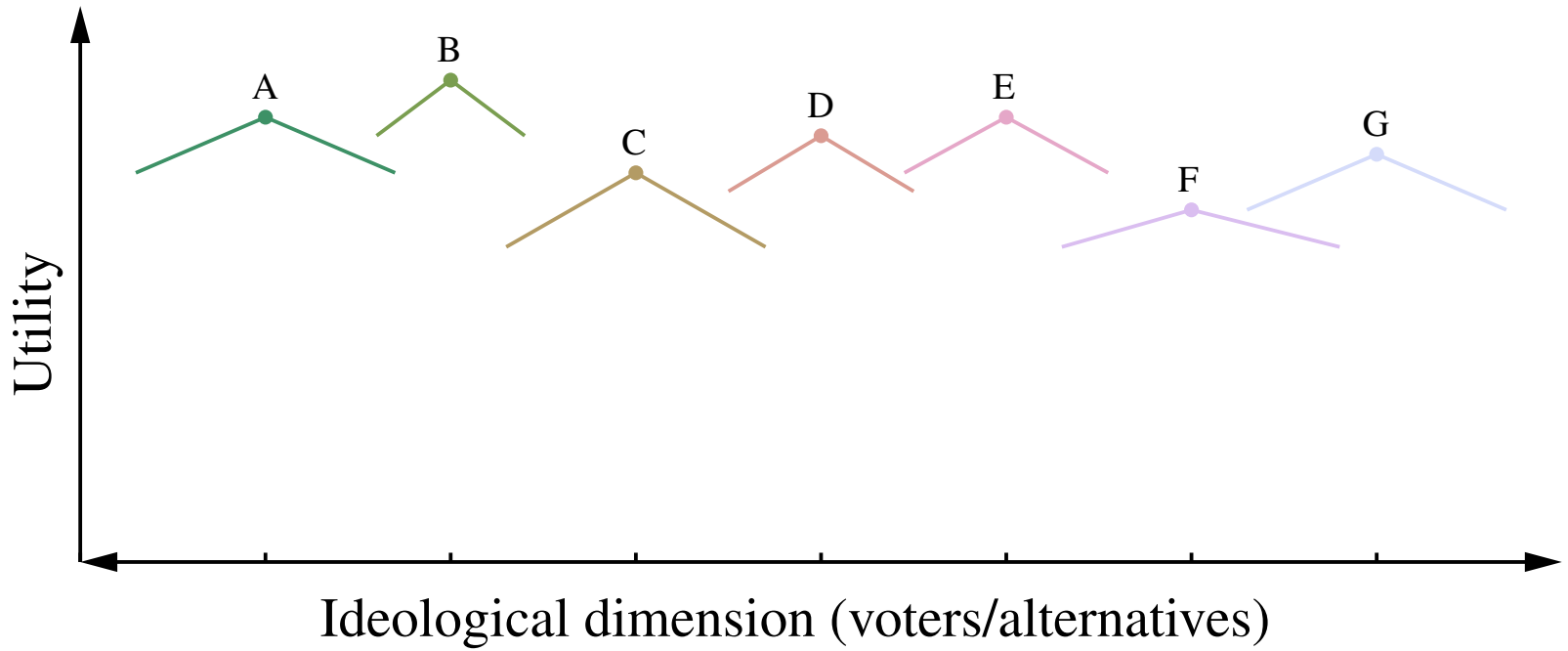
In a committee of *n* members, who are all endowed with single-peaked preferences, the proposal submitted by the median voter is a Condorcet-winner: it prevails over any other proposal in a paired comparison.



Median voter theorem (strong version)

Black (1948)

In a committee of *n* members, who are all endowed with single-peaked and symmetric preferences the proposal supported by the median voter always wins.



Measuring voting power

What is the influence of each voter in a voting body?

- Shapley, L. S., & Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. American Political Science Review, 48(3), 787–792.
- Penrose, L. S. (1946). The elementary statistics of majority voting. Journal of the Royal Statistical Society, 109(1), 53–57.
- Banzhaf, J. F. (1965). Weighted voting doesn't work: A mathematical analysis. Rutgers Law Review, 19, 317–343.

$\therefore$ Power measures.

Literature on Convex voting

- Edelman and Jamison (1985) - idea of convex voting games
- Bilbao, Jiménez and López (1998) - Banzhaf-index in convex voting games
- Bilbao and Edelman (2000) - Shapley-value in convex voting games
- **Kóczy and Sziklai (2015) - Papal conclave**
- **Fertő, Kovács, Kóczy and Sziklai (2020) - Agricultural Committee of the EP**

Voting games

- A set of voters $N = \{1, 2, ...i, ...n\}$
- Coalitions: subsets of voters $C \subseteq N$
- Winning ($\mathcal{W}$) and losing coalitions
- A cooperative game $v : 2^N \to \{0, 1\}$, where $v(C) = 1$ iff winning.

The usual assumptions:

- $N \in \mathcal{W}$, $\emptyset \notin \mathcal{W}$
- if $C, D \in \mathcal{W}$, $C \cap D \neq \emptyset$
- if $C \in \mathcal{W}$ and $D \supset C$, then $D \in \mathcal{W}$

Critical voters

Definition - Critical voters

We say that voter i is **critical** for coalition $C \subseteq N$, if $C \in \mathcal{W}$, but $C \setminus i \notin \mathcal{W}$ (that is, it can turn a winning coalition into a losing one).

Definition - Banzhaf-index

Let us denote by $\mathcal{W}_i \subseteq \mathcal{W}$ those winning coalitions for which i is critical, and let $\eta_i = |\mathcal{W}_i|$. Then Banzhaf-index can be calculated as follows

$$\beta_i = \frac{\eta_i}{\sum_{j \in N} \eta_j}.$$

Voting when the ideology space has more dimension

Assumption - Convexity

Suppose the committee has to put together a bill. The draft is being amended until it gets majority support. If the bill is acceptable for two voters occupying the opposite sides of one ideological dimension (say a left-wing and a right-wing voter), then it will be acceptable for every voter in between as long as they have no objection related to other ideological dimensions.

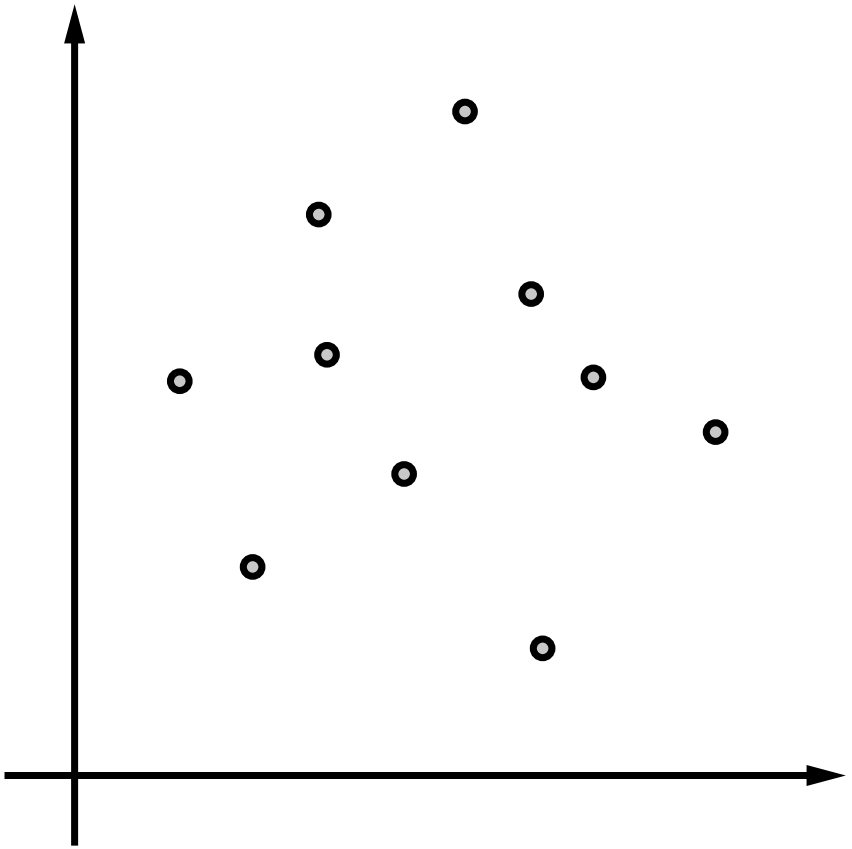
Corollary

Only those voters are critical, who are on the very edge of supporting the bill. In the ideology space they are located on the boundary of the coalition.

Preferences: 2 dimensions

Voters are distributed over a 2-dimensional policy space.

Assume convexity in both directions.



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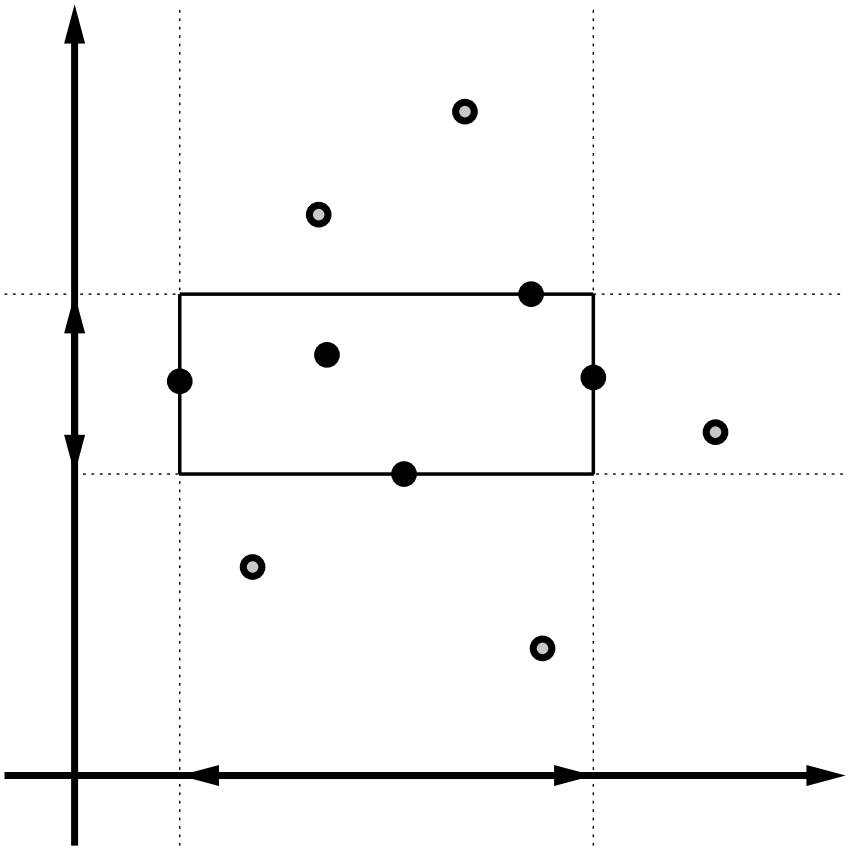

Convex

Concave.

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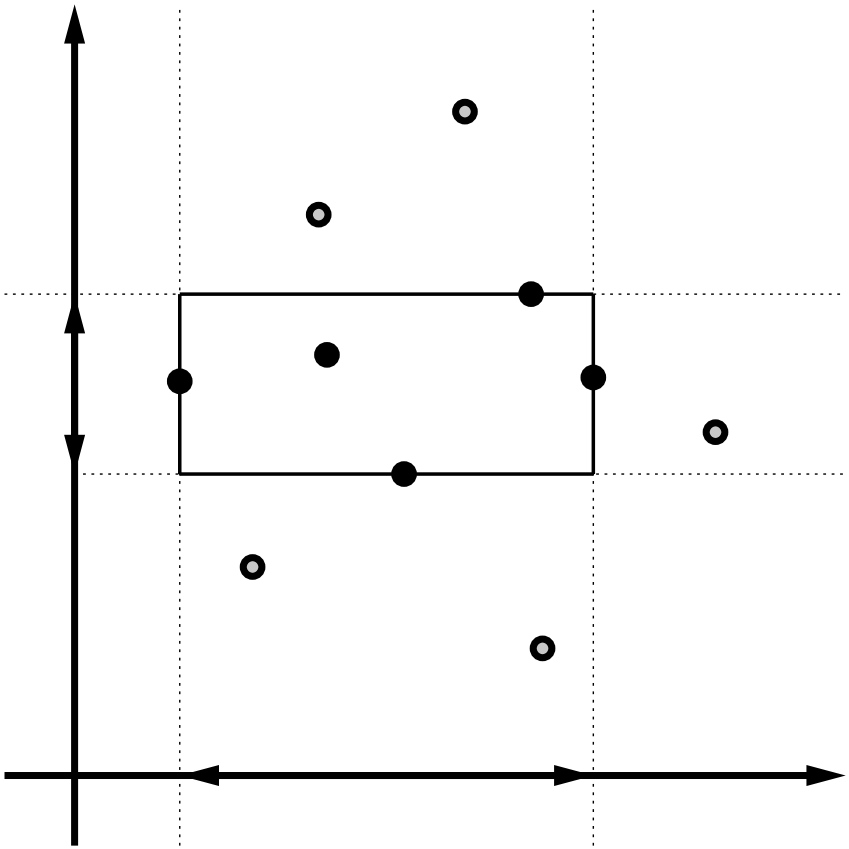

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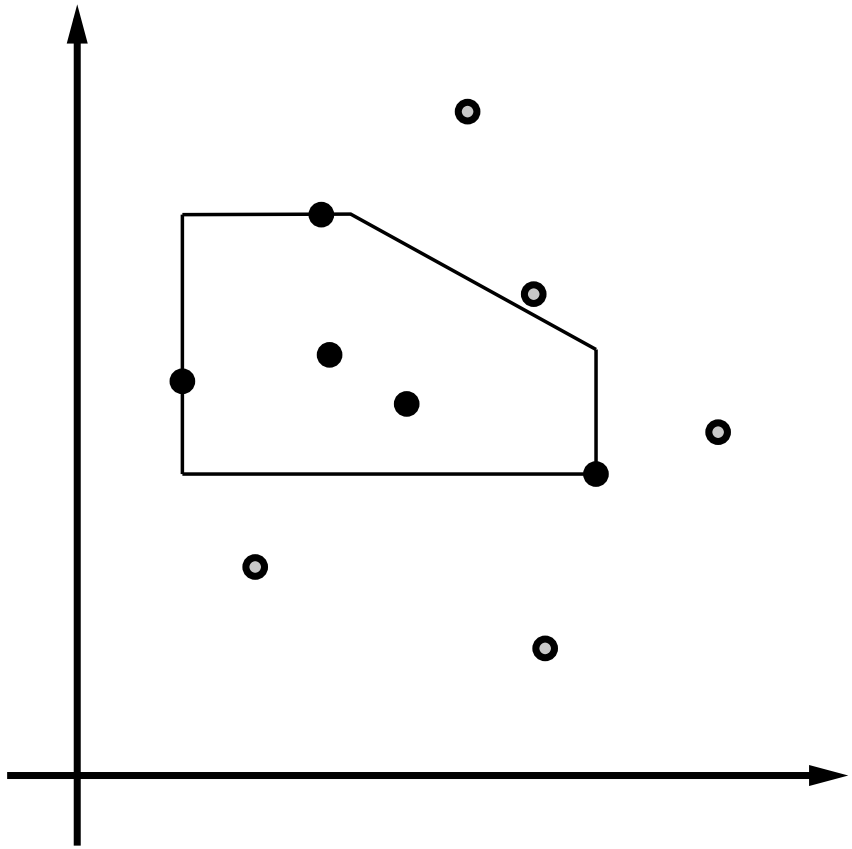

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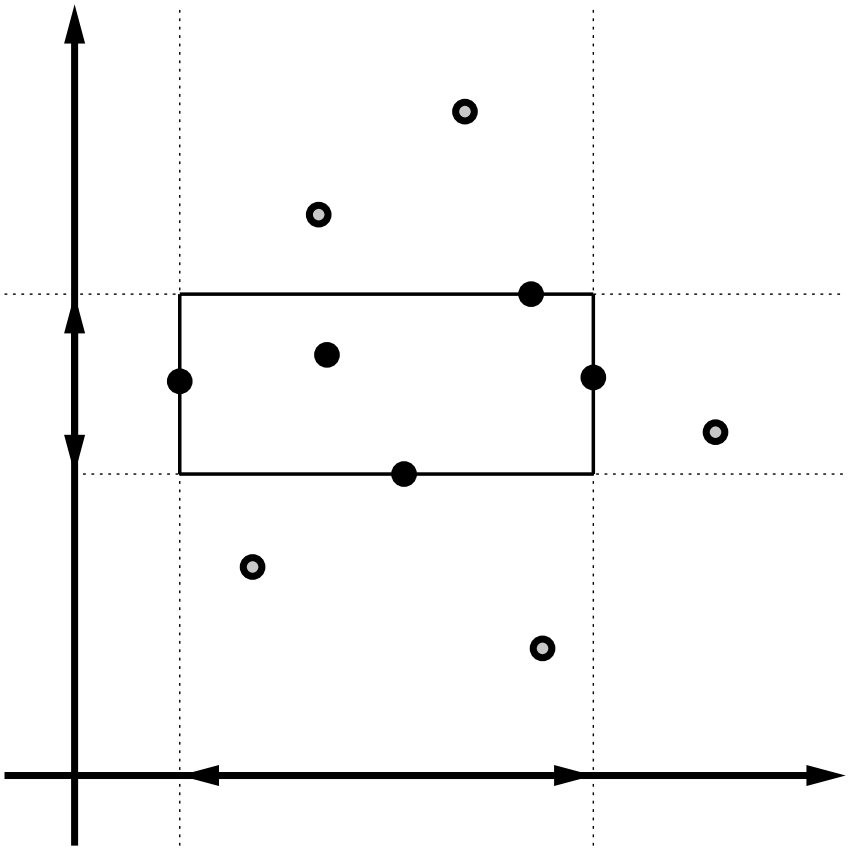

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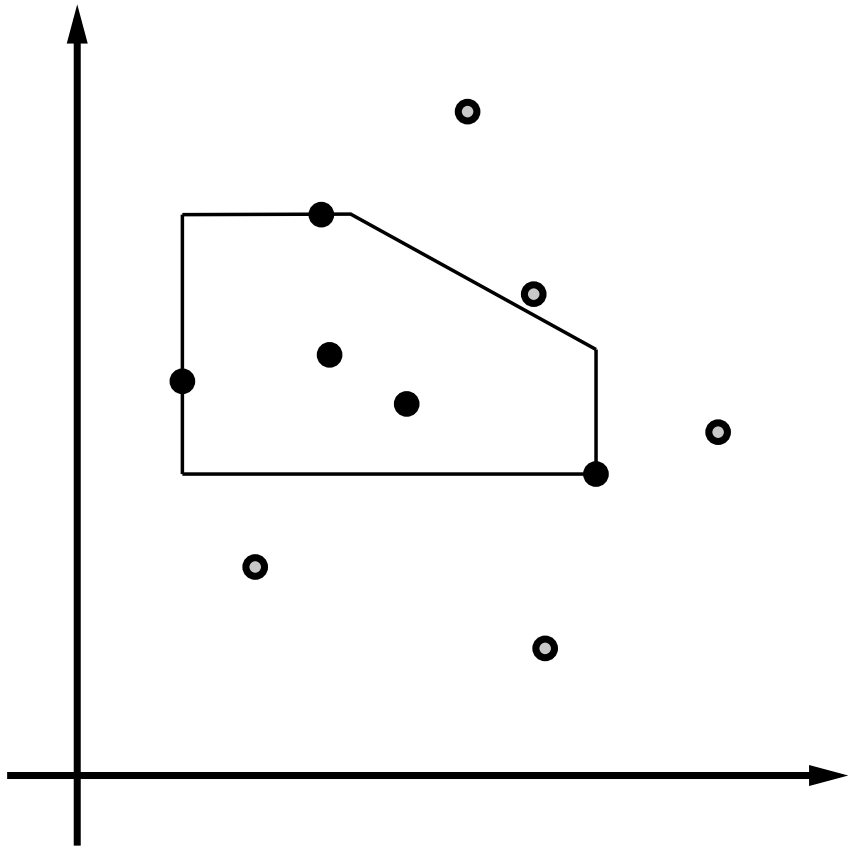
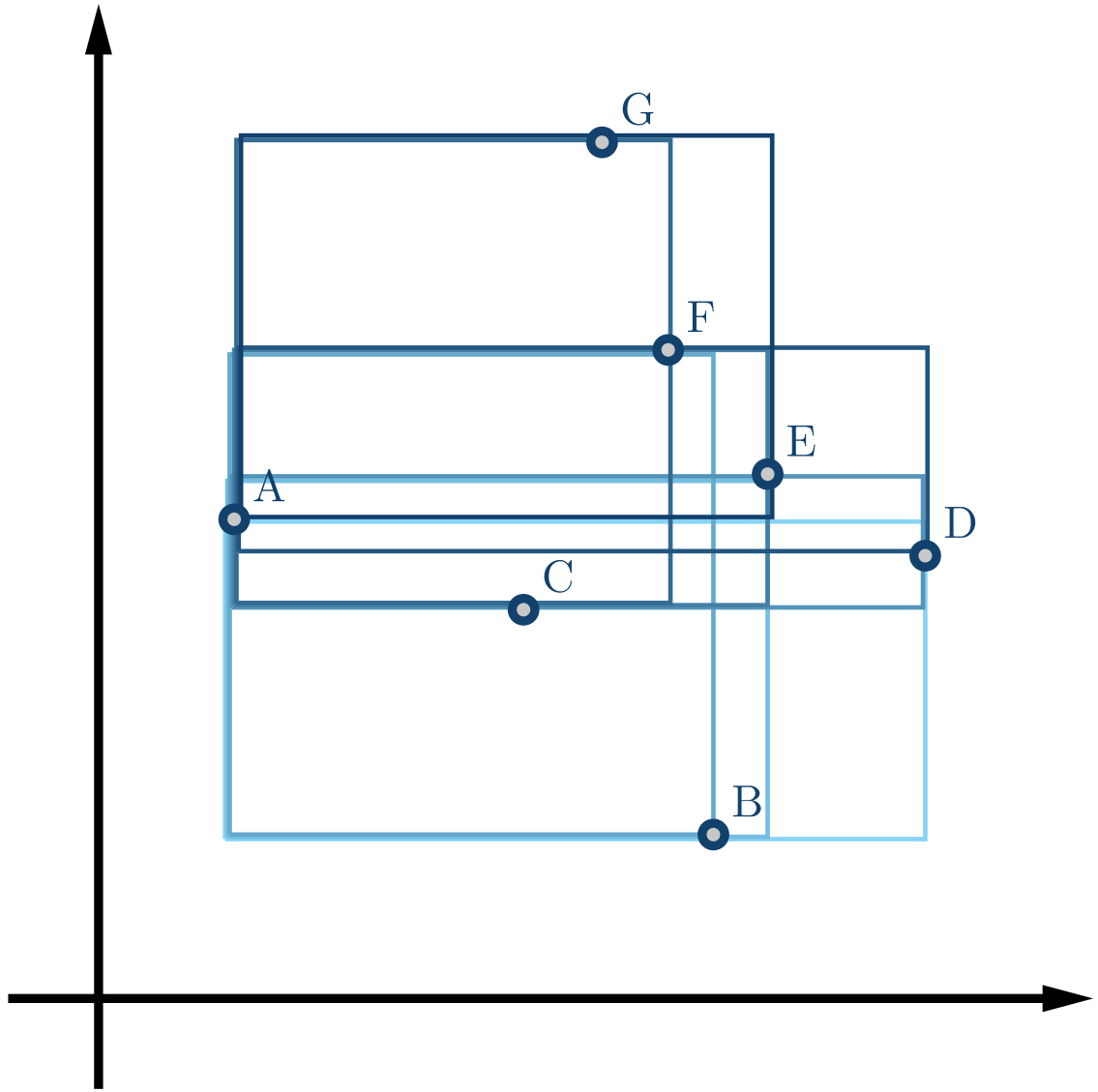


Concave.

Power: 2 dimensions

- symmetry except for location
- only MWCs (size q)
- convexity $\rightarrow$ spanned rectangles
- critical points are on the borderline
- power: how often is a voter on the borderline of such a q-rectangle?

Example: 7 voters; 4-rectangles over A.



Papal elections – 2013

The head of the Catholic church is elected

- by cardinals who are below 80 yr;
- by 2/3 majority (requiring 77 of 115 votes in 2013).
- in a *conclave* (anonymous voting, behind closed doors).

There are no parties, the voters are well-known. Two issues:

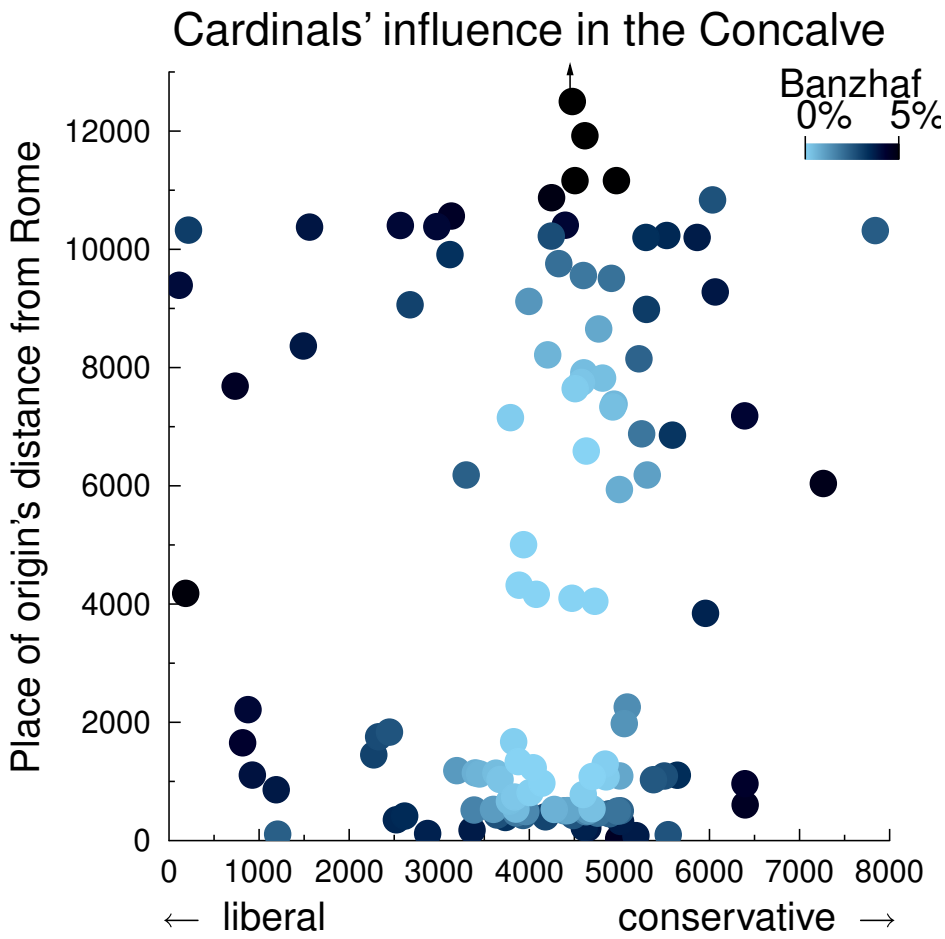
- Italian or overseas
- liberal or conservative

Oddsmakers' blindness

UPDATE: Betting on the next pope: Oddsmakers blow it with top 15 pope picks

Cardinal Peter Turkson of Ghana. WIKIMEDIA COMMONS

Influential cardinals I.



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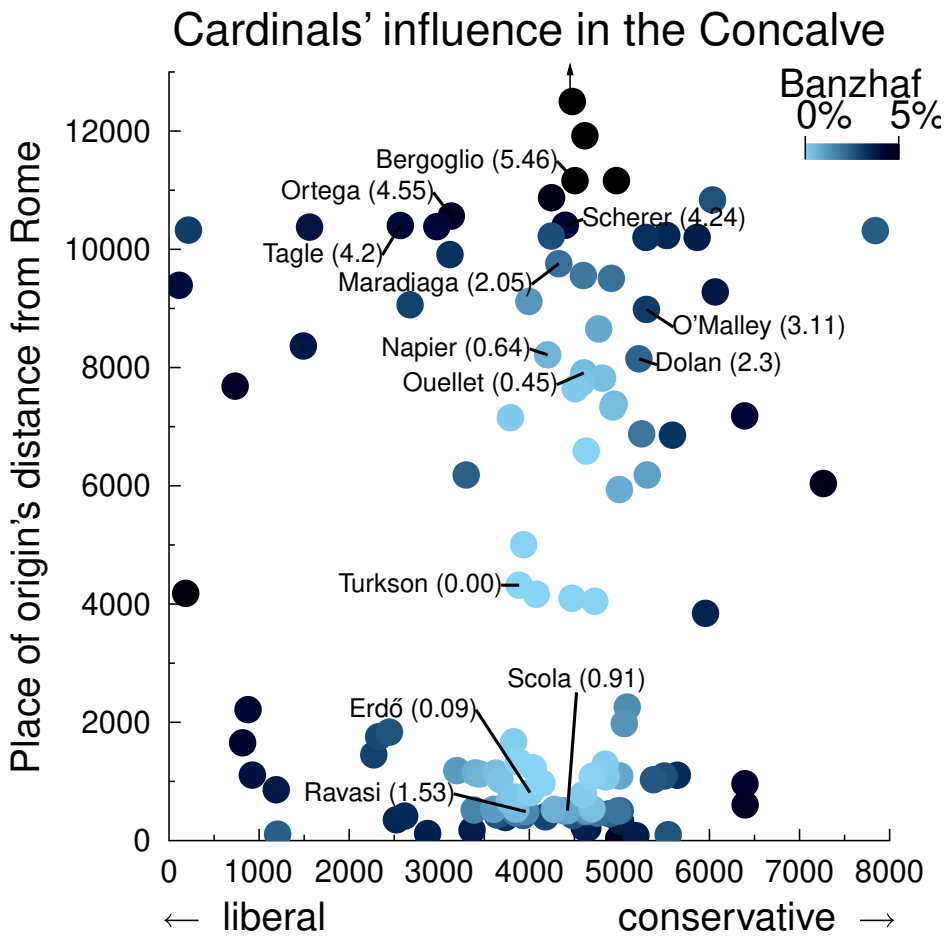


Influential cardinals II.

rang	bíboros	c_y (km)	c_x (bp)	η_i
1	George Pell	15902	4477	599
2	Francisco Javier Errázuriz Ossa	11921	4617	572
3	Jorge Bergoglio	11162	4507	546
4	Leonardo Sandri	11162	4967	536
5	Théodore-Adrien Sarr	4181	185	517
6	Juan Luis Cipriani Thorne	10873	4247	497
7	Telesphore Placidus Toppo	6039	7264	481
8	Agostino Vallini	34	4992	461
8	Jean-Pierre Ricard	603	6396	461
10	James Michael Harvey	7686	734	459

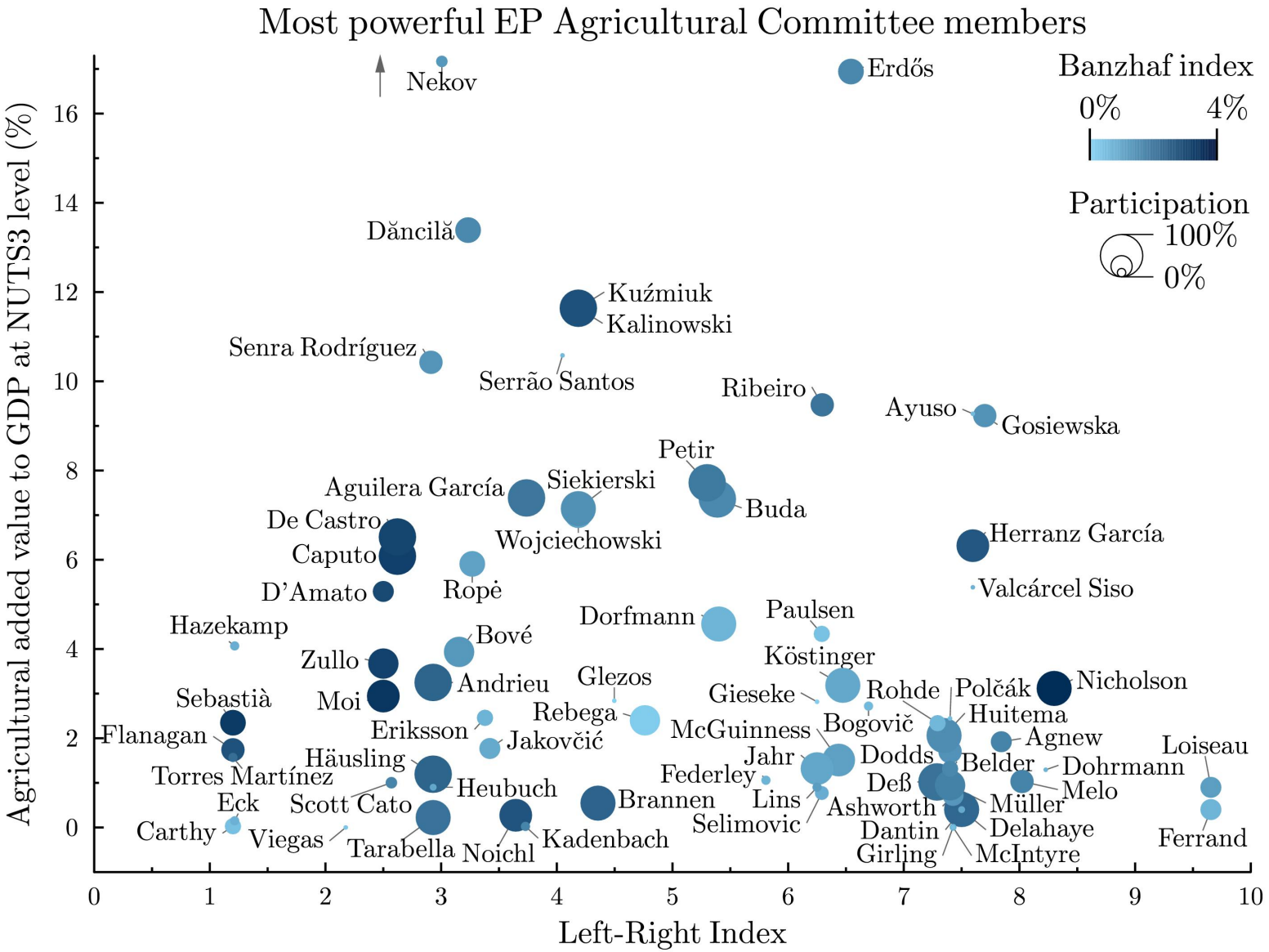
Some lessons

- Power indices perform much better than oddsmakers (Bergoglio (Pope Francis) 3/115, Peter Turkson 107-115/115)
- The median voter seems to be a null-player or has a marginal influence.
- Voters with relatively extreme positions in one dimension and moderate positions in the other one are the influential members.

COMAGRI

- EP Committees are the legislative backbones of the EU
- We analysed the 2014-19 legislative period of the agricultural committee of the EP (COMAGRI).
- Total of 71 MEPs (46 committee members and 25 substitutes).
- Due to substitution, the number of possible committee configurations is astronomical $\longrightarrow$ we used a Monte Carlo simulation to estimate the Banzhaf-index.
- The probability of substitution was derived from attendance data.
- The ideological space are spanned by the classical left/right dimension (derived from party affiliations) and agricultural affinity (the marginal contribution of the agricultural sector to GDP at the MEP's place of birth).

COMAGRI ideological space



Some lessons

- Substitutes may have surprisingly high influence, especially since invitations do not seem to be random.
- Being reliable, in the sense of not missing many sessions, can increase influence greatly.
- Results are in line with literature, that report that rapporteurs and members in formal positions have high influence.
- **Median voter is powerless.**
- **Again: Voters with relatively extreme positions in one dimension and moderate positions in the other one are the influential members.**

Plan of attack

Both case studies affirm that in two dimensions the influence of the median voter disappears. Does this happen by chence or is it a theoretical necessity? We approach the problem from three direction

- Simulation on a grid
- Walk in the ideological space
- Combinatorial analysis (work in progress)

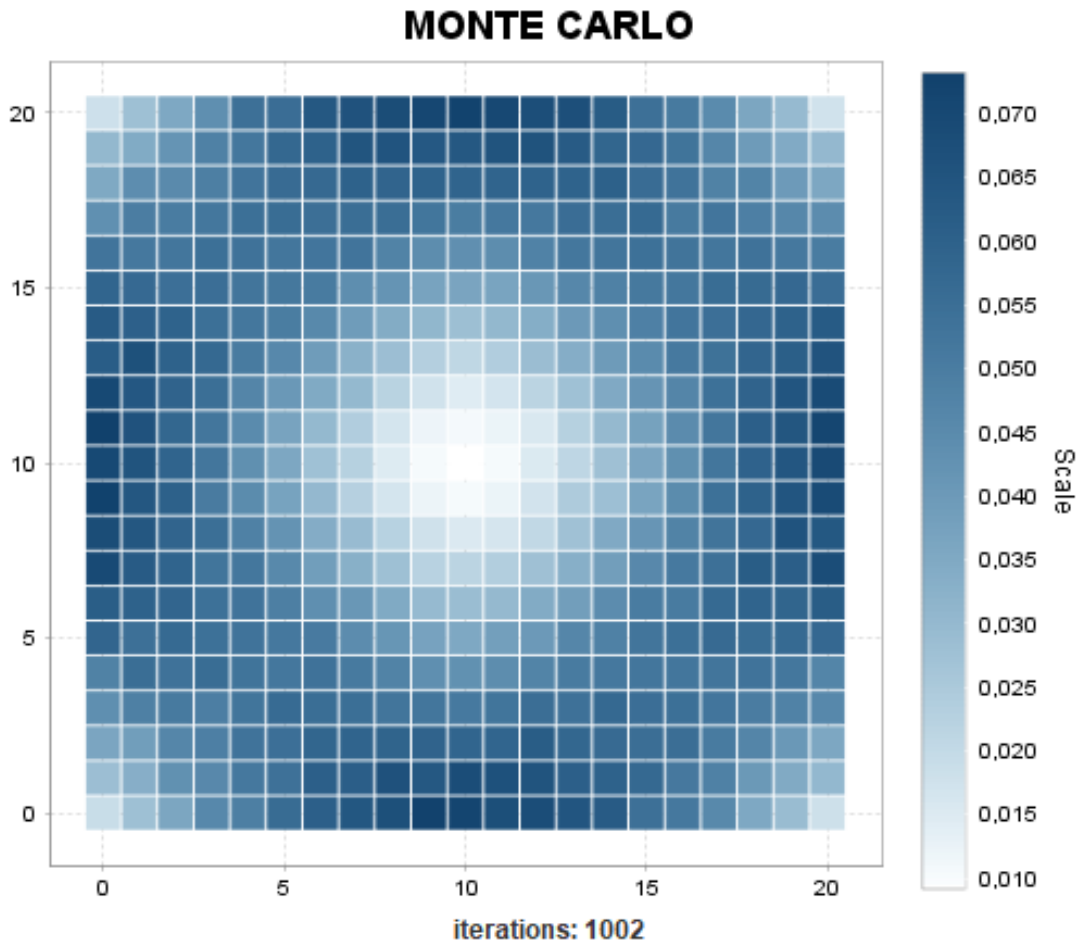
Simulation of a grid

We assume that the each member occupy a unique point in the ideological space (no coordinates coincide). Thus we can sort them into increasing order along each dimension. As a consequence

- members can be put on a grid where
- each row and each column contains exactly one voter (like rooks on a chessboard).

We generate a committee randomly, then compute the Banzhaf-index. We assign the computed values to their position on the grid. We iterate the process until we obtain a sufficiently large sample for each point of the grid. Sample average shows how influential is a given position on a grid.

$n = 21,\ q = 11$ (simple majority)



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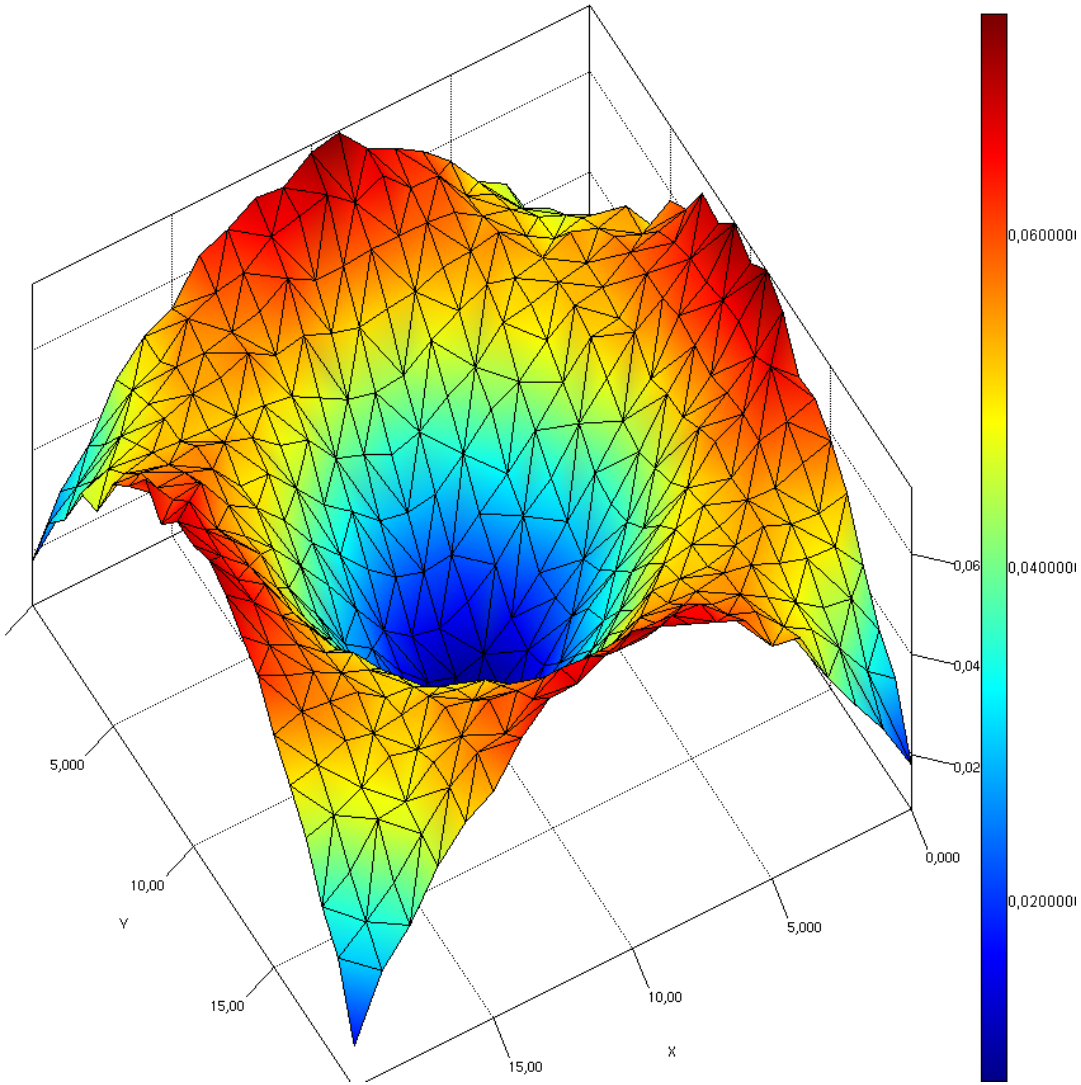
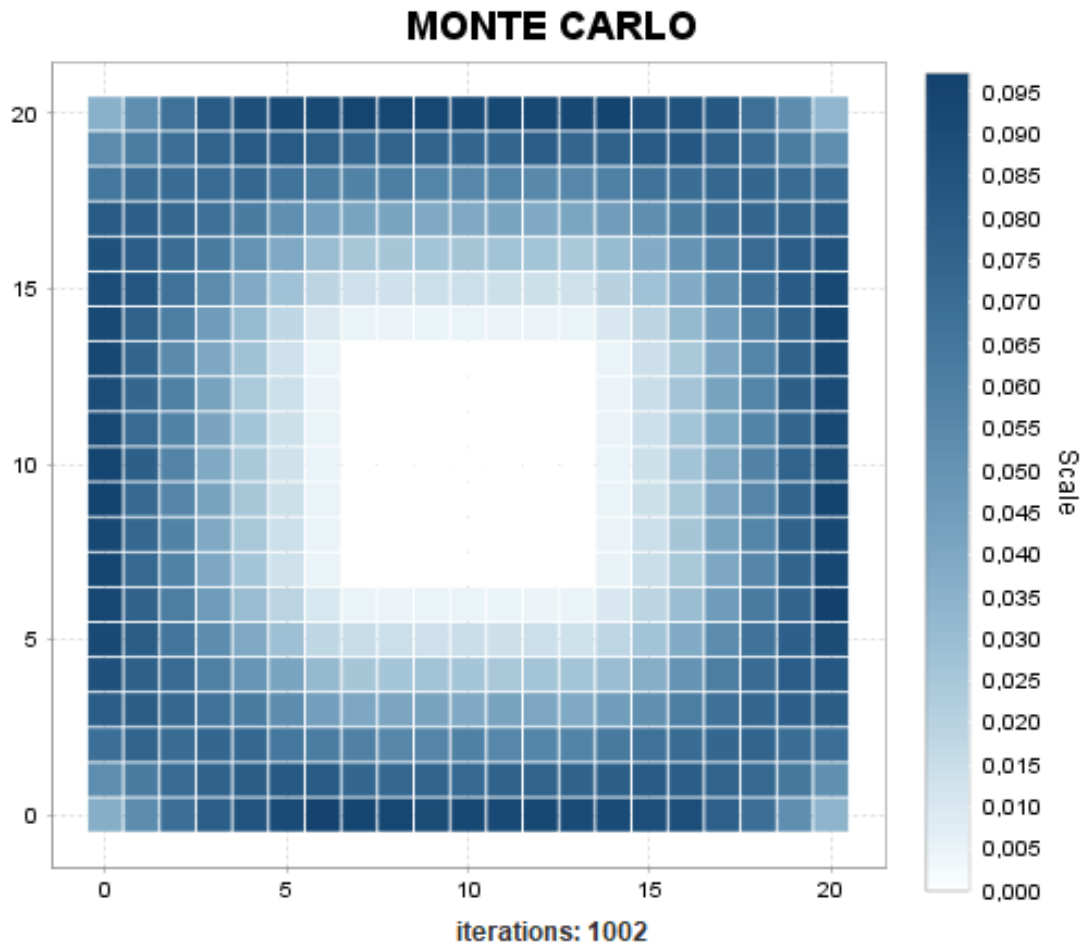


$n = 21,\ q = 11$ (qualified majority)



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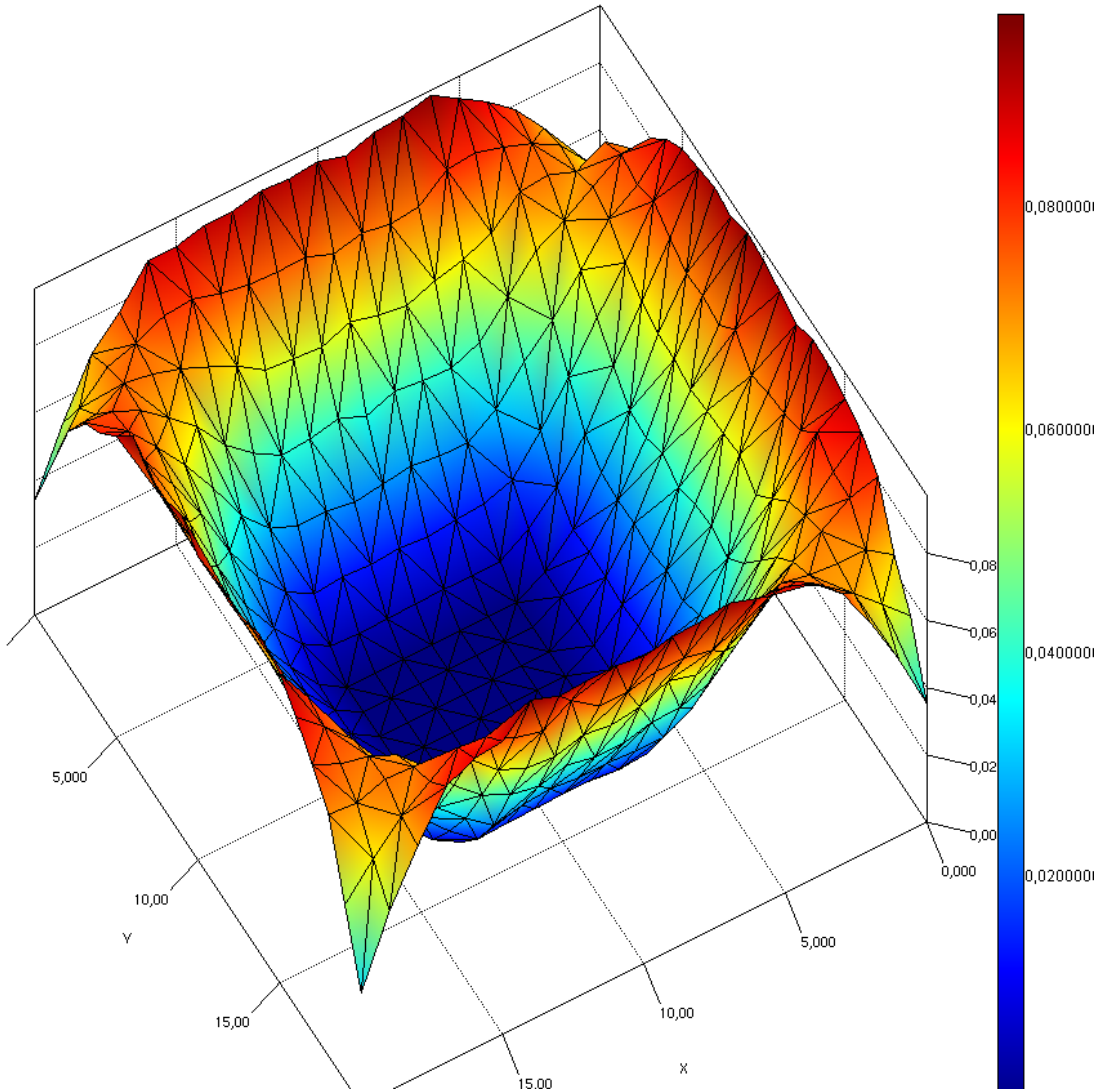
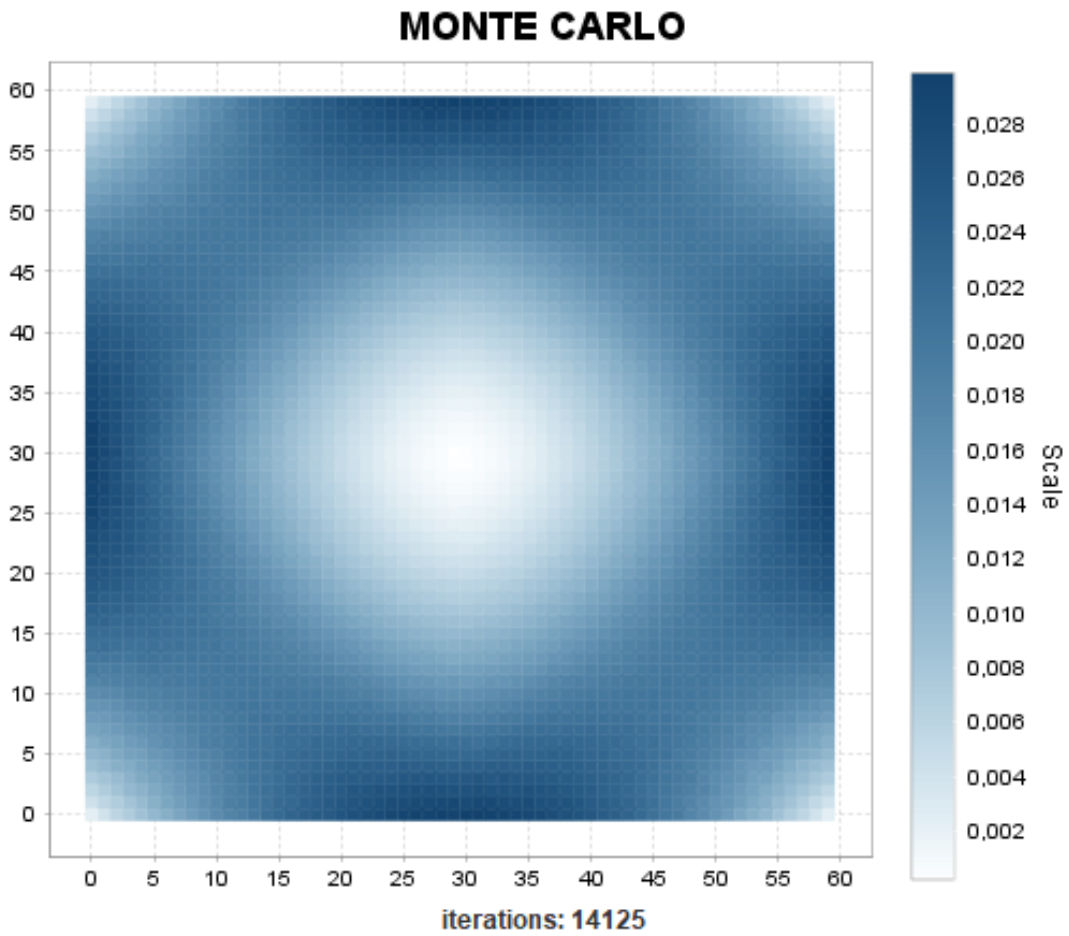


$n = 60,\ q = 31$ (simple majority)



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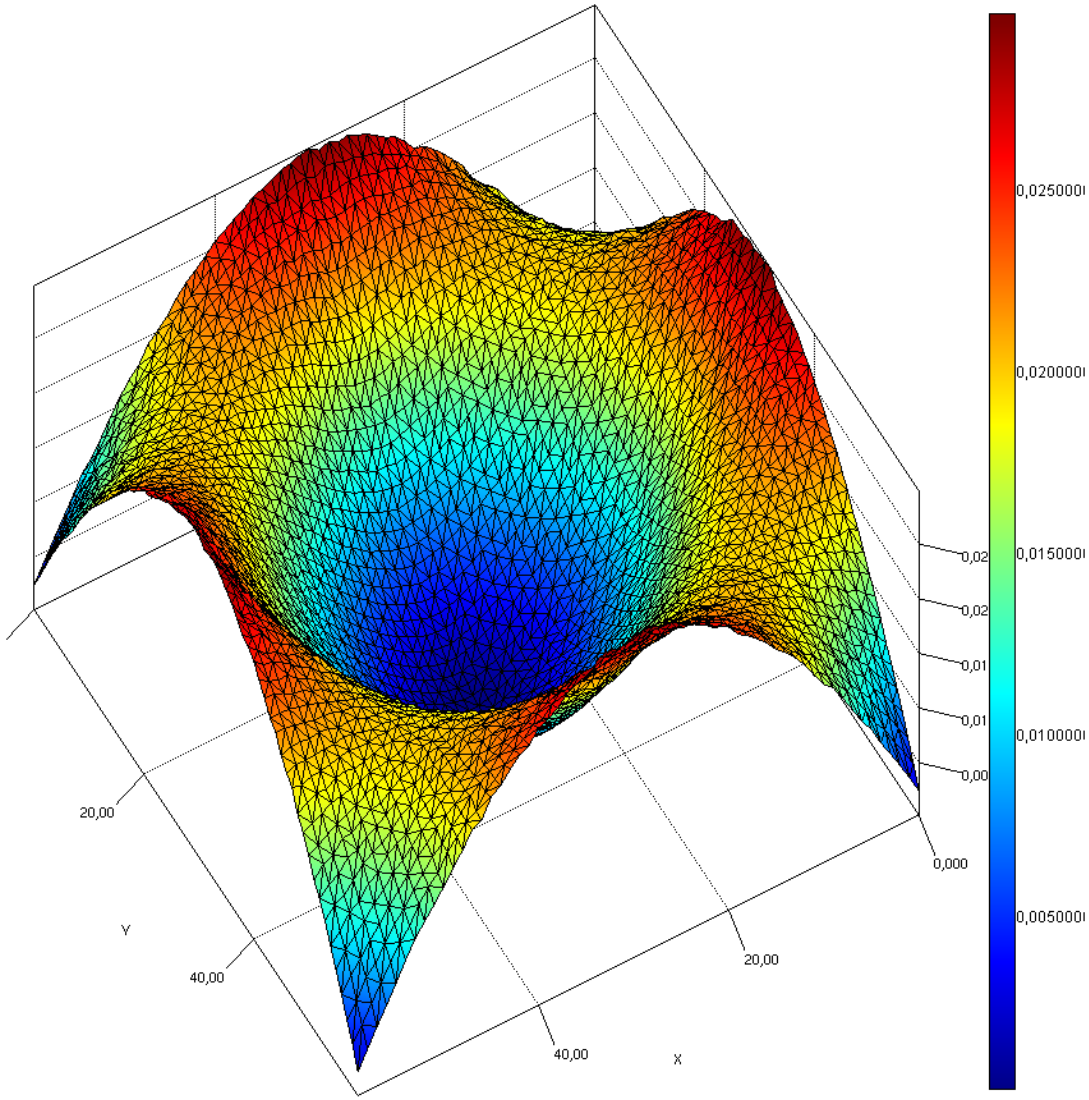
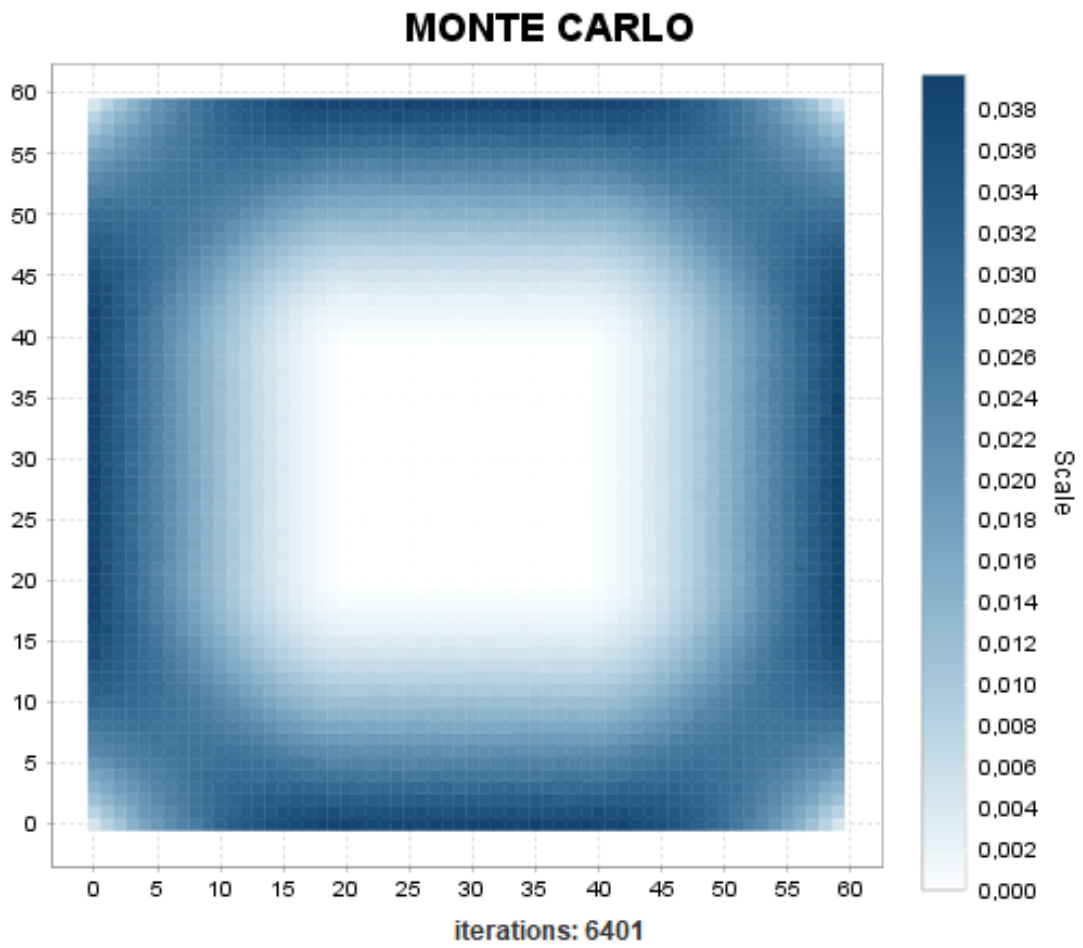


$n = 60,\ q = 41$ (qualified majority)



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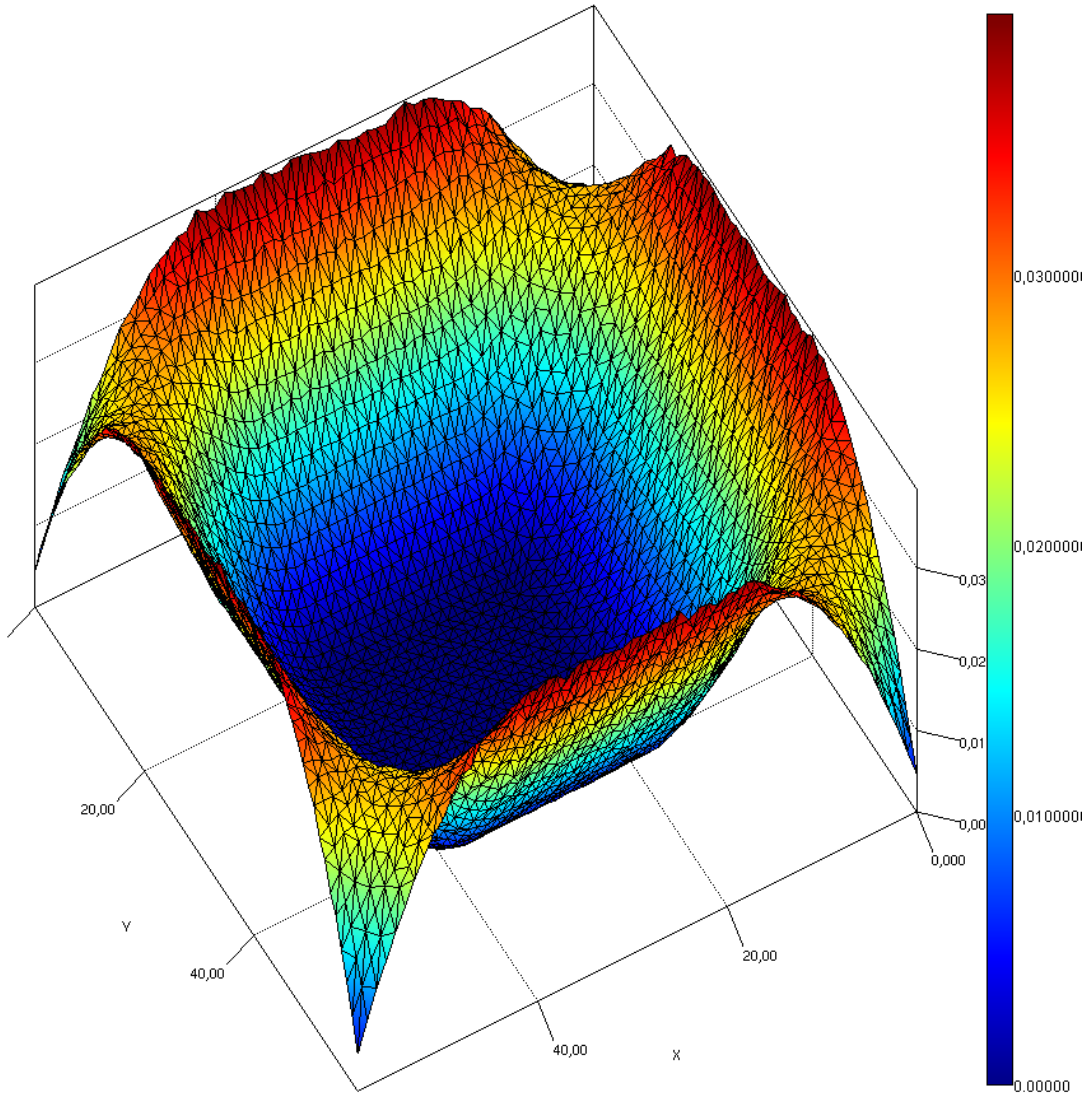
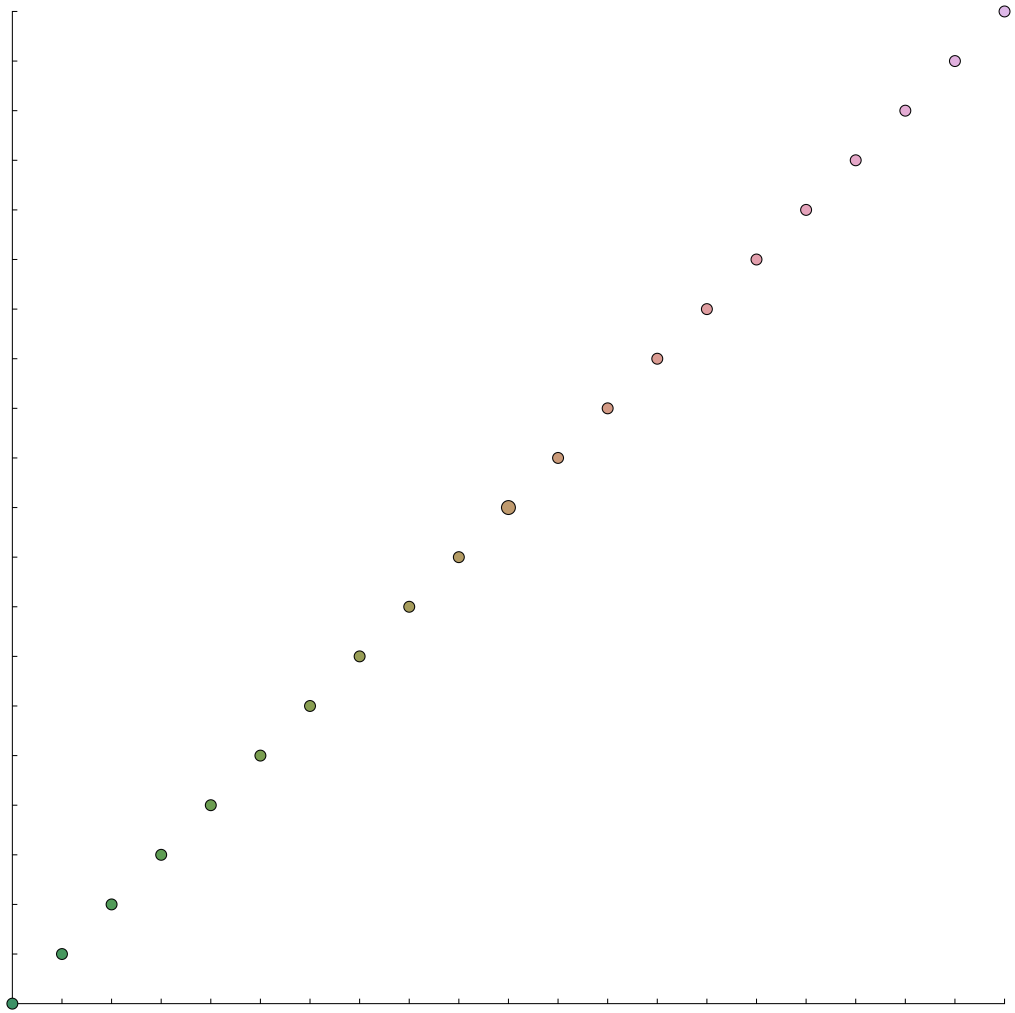


Walk in the ideological space

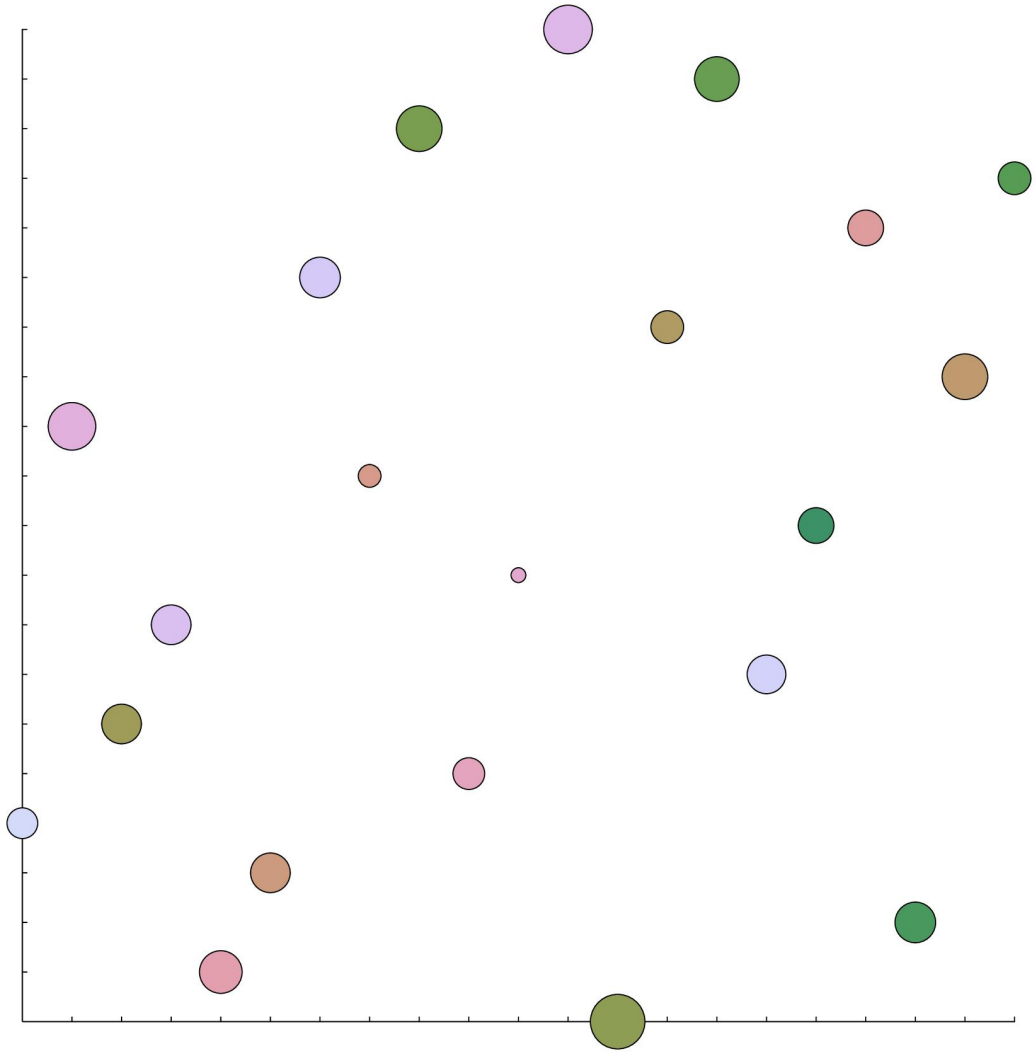
Suppose that the political debate is dominated by one issue for a long time. Voters are spread across the ideological dimension from one extreme to the other. What happens if a new issue emerges? We expect voters to position themselves in the ideological space to maximize their influence. This is equivalent with saying that they want to be critical to as many coalitions as possible.

- Similarly as before we work with a grid.
- In each step of the simulation a voter is chosen at random who may tade places with one of its neighbours.
- Since the grid represent the relative positions of the voters, this switch symbolize that the voter goes one step further than its neighbour.

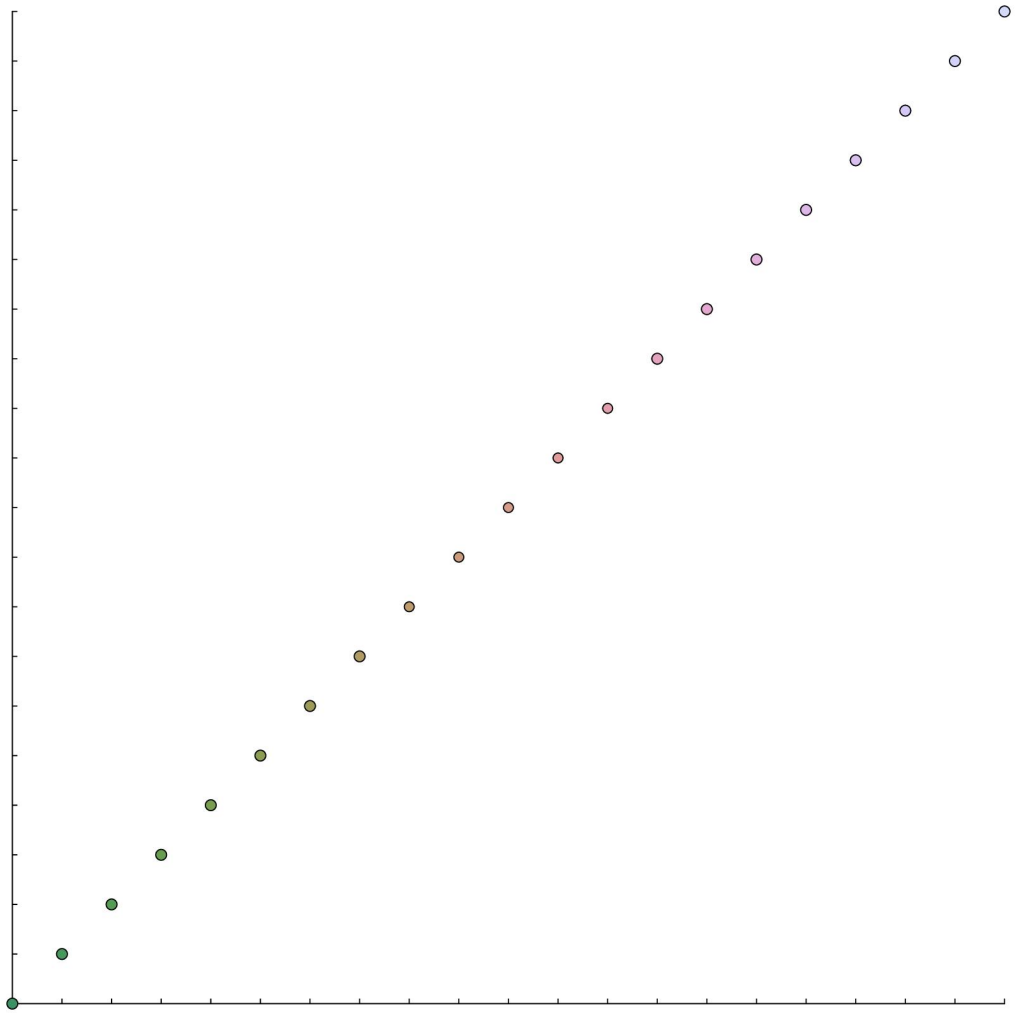
$n = 21,\ q = 11$ start from 1 dimension



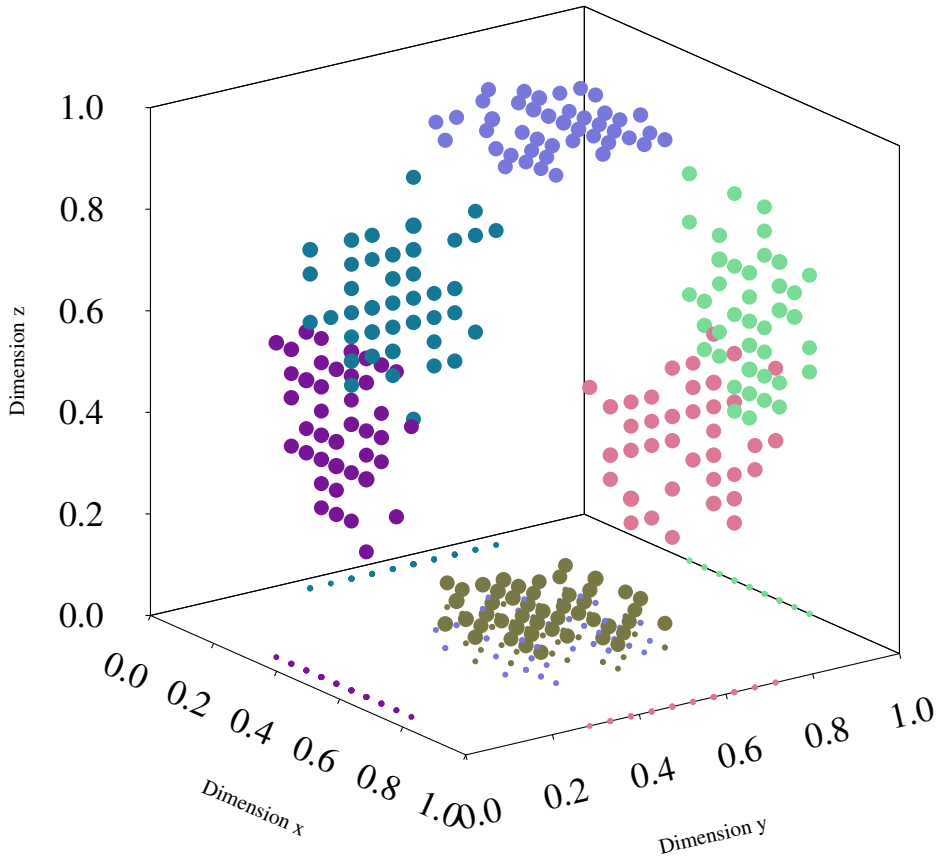
$n = 21,\ q = 11$ start from 2 dimension



$n = 21,\ q = 14$ start from 1 dimension



Walk in 3 dimension



Lessons

- The influence of the median voter is zero if there are more than one issues.
- The most influential voters are those who take extreme position in one ideological dimension and moderate positions on the others.
- Increasing the quota makes the centrum less influential (see EU's foreign policy).
- Convergence is fast independently of the starting setup.
- Working with absolute positions instead of relative positions does not change the outcome.

Research questions

- can the model explain the polarization of the political debate?
- What happens if dimensions correlate?
- Combinatorial proof instead of simulation

Thank you for your attention!