A cooperative game approach to integrated health care

Guillaume Sekli

Games and Optimization - Saint-Etienne

April 2022

Introduction

We observe an increase of patients who have chronic disease (Hackbarth et al., 2008) in a context of bad prospections concerning the increasing of old population (OECD, 2017).

The question of how taking in charge chronic patient become more and more relevant in health systems which are very competitive and fragmentated (Brekke et al., 2021).

A “credible solution for the future“ is the implementation of integrated healthcare through a bundled payment (Porter and Lee, 2013)

Introduction

We observe an increase of patients who have chronic disease (Hackbarth et al., 2008) in a context of bad prospections concerning the increasing of old population (OECD, 2017).

The question of how taking in charge chronic patient become more and more relevant in health systems which are very competitive and fragmentated (Brekke et al., 2021).

A "credible solution for the future" is the implementation of integrated healthcare through a bundled payment (Porter and Lee, 2013)

Introduction

We observe an increase of patients who have chronic disease (Hackbarth et al., 2008) in a context of bad prospections concerning the increasing of old population (OECD, 2017).

The question of how taking in charge chronic patient become more and more relevant in health systems which are very competitive and fragmentated (Brekke et al., 2021).

A “credible solution for the future“ is the implementation of integrated healthcare through a bundled payment (Porter and Lee, 2013)

Introduction

Medical act(s) (Rapport Véran, 2017)

Heavy surgical procedures or acute medical care that requires short-stay care, outpatient care, or home care.

$\Rightarrow$ **The patient pays all individual prices**

Integrated health care (Rapport Véran, 2017)

A set of care provided for a given health condition, during a given period of time and by all the health professionals involved in the care. The care or health pathway includes prevention or health education activities, coordination, and patient support for care.

$\Rightarrow$ **The patient pays a unique fee**

Introduction

Medical act(s) (Rapport Véran, 2017)

Heavy surgical procedures or acute medical care that requires short-stay care, outpatient care, or home care.
$\Rightarrow$ **The patient pays all individual prices**

Integrated health care (Rapport Véran, 2017)

A set of care provided for a given health condition, during a given period of time and by all the health professionals involved in the care. The care or health pathway includes prevention or health education activities, coordination, and patient support for care.
$\Rightarrow$ **The patient pays a unique fee**

Introduction

Medical act(s) (Rapport Véran, 2017)

Heavy surgical procedures or acute medical care that requires short-stay care, outpatient care, or home care.
$\Rightarrow$ **The patient pays all individual prices**

Integrated health care (Rapport Véran, 2017)

A set of care provided for a given health condition, during a given period of time and by all the health professionals involved in the care. The care or health pathway includes prevention or health education activities, coordination, and patient support for care.
$\Rightarrow$ **The patient pays a unique fee**

Introduction

Medical act(s) (Rapport Véran, 2017)

Heavy surgical procedures or acute medical care that requires short-stay care, outpatient care, or home care.
$\Rightarrow$ **The patient pays all individual prices**

Integrated health care (Rapport Véran, 2017)

A set of care provided for a given health condition, during a given period of time and by all the health professionals involved in the care. The care or health pathway includes prevention or health education activities, coordination, and patient support for care.
$\Rightarrow$ **The patient pays a unique fee**

Introduction

There are some advantages to integrated healthcare system (HCAAM, 2015; Stokes et al., 2018).

Many experimentations in a lot of countries with good results (HCAAM, 2015; Busse and Stahl, 2014; Struijs and Baan, 2011; de Bakker et al., 2012).

Bundled payment in integrated healthcare are surveyed by Rocks et al. (2020).

Introduction

There are some advantages to integrated healthcare system (HCAAM, 2015; Stokes et al., 2018).

Many experimentations in a lot of countries with good results (HCAAM, 2015; Busse and Stahl, 2014; Struijs and Baan, 2011; de Bakker et al., 2012).

Bundled payment in integrated healthcare are surveyed by Rocks et al. (2020).

Introduction

There are some advantages to integrated healthcare system (HCAAM, 2015; Stokes et al., 2018).

Many experimentations in a lot of countries with good results (HCAAM, 2015; Busse and Stahl, 2014; Struijs and Baan, 2011; de Bakker et al., 2012).

Bundled payment in integrated healthcare are surveyed by Rocks et al. (2020).

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}$, $c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \qquad (1)$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}$, $c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \tag{1}$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}$, $c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \tag{1}$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}, c_q \in N$ is identified by the provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \qquad (1)$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}, c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \tag{1}$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Notations

A bundled payment B is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, ..., n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by i.
- $C = (c_1, ..., c_k)$ a chain modeling the recovery path where each service $q \in \{1, ..., k\}, c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1,...,k\}} p_{c_q} > F \qquad (1)$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

$$\sum_{i \in N} x_i = F$$

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:
$$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$
- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:

$$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:

  $$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:

$$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:

$$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Example : Lungs cancer

- $N = \{P, S, H\}$ with P the **practician**, S the **specialist** and H **the hospital**
- $p_P = 25, p_S = 45, p_H = 110.$
- $C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7)$ is represented by:
$$H \longrightarrow H \longrightarrow P \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$
- The total cost the patient should have paid is: $110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535$
- Suppose a fee $F = 400$.

Bankruptcy games

The problem B can be apprehended by means of cooperative game theory and can be inspired by bankruptcy games literature (O'Neill, 1982; Aumann and Maschler, 1985):

- The total estate to share among healthcare professionals is F
- The claimants are healthcare professionals
- One possibility is to consider that the claims are the total turnover

Bankruptcy games

The problem B can be apprehended by means of cooperative game theory and can be inspired by bankruptcy games literature (O'Neill, 1982; Aumann and Maschler, 1985):

- The total estate to share among healthcare professionals is F
- The claimants are healthcare professionals
- One possibility is to consider that the claims are the total turnover

Bankruptcy games

The problem B can be apprehended by means of cooperative game theory and can be inspired by bankruptcy games literature (O'Neill, 1982; Aumann and Maschler, 1985):

- The total estate to share among healthcare professionals is F
- The claimants are healthcare professionals
- One possibility is to consider that the claims are the total turnover

Bankruptcy games

The problem B can be apprehended by means of cooperative game theory and can be inspired by bankruptcy games literature (O'Neill, 1982; Aumann and Maschler, 1985):

- The total estate to share among healthcare professionals is F
- The claimants are healthcare professionals
- One possibility is to consider that the claims are the total turnover

Bankruptcy games

There are two original features:

- The order of interventions during the process
- The possibility for an healthcare professional to act several times during the process

There are bankruptcy games which taking into account the position of players (Ansink and Weikard, 2012) but without the possibility to be in more than one position.

Bankruptcy games

There are two original features:

- The order of interventions during the process
- The possibility for an healthcare professional to act several times during the process

There are bankruptcy games which taking into account the position of players (Ansink and Weikard, 2012) but without the possibility to be in more than one position.

Bankruptcy games

There are two original features:

- The order of interventions during the process
- The possibility for an healthcare professional to act several times during the process

There are bankruptcy games which taking into account the position of players (Ansink and Weikard, 2012) but without the possibility to be in more than one position.

Bankruptcy games

There are two original features:

- The order of interventions during the process
- The possibility for an healthcare professional to act several times during the process

There are bankruptcy games which taking into account the position of players (Ansink and Weikard, 2012) but without the possibility to be in more than one position.

The chain and the turnover

The healthcare professionnals have the possibility to act one time or more than once. The set of all events for a player $i \in N$ is a correspondance $N \longrightarrow \{1, ..., k\}$ that associates to each $i \in N$ one or more positions in the chain C. This is done by the inverse function $C^{-1}(i)$ defined as:

$$C^{-1}(i) = \Big\{ q \in \{1, ..., k\} : c_q = i \Big\}, \quad \forall i \in N.$$

The total turnover involving i is:

$$\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.$$

This turnover can be interpreted as the legitimate claim of health professional i or its bargaining power when sharing F, which refers naturally to the bankruptcy approach.

The chain and the turnover

The healthcare professionnals have the possibility to act one time or more than once. The set of all events for a player $i \in N$ is a correspondance $N \longrightarrow \{1, ..., k\}$ that associates to each $i \in N$ one or more positions in the chain C. This is done by the inverse function $C^{-1}(i)$ defined as:

$$C^{-1}(i) = \Big\{ q \in \{1, ..., k\} : c_q = i \Big\}, \quad \forall i \in N.$$

The total turnover involving i is:

$$\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.$$

This turnover can be interpreted as the legitimate claim of health professional i or its bargaining power when sharing F, which refers naturally to the bankruptcy approach.

The chain and the turnover

The healthcare professionnals have the possibility to act one time or more than once. The set of all events for a player $i \in N$ is a correspondance $N \longrightarrow \{1, ..., k\}$ that associates to each $i \in N$ one or more positions in the chain C. This is done by the inverse function $C^{-1}(i)$ defined as:

$$C^{-1}(i) = \Big\{ q \in \{1, ..., k\} : c_q = i \Big\}, \quad \forall i \in N.$$

The total turnover involving i is:

$$\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.$$

This turnover can be interpreted as the legitimate claim of health professional i or its bargaining power when sharing F, which refers naturally to the bankruptcy approach.

The maximal chain

For each $S \subseteq N$, let $C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i)$,

Maximal chain

The **maximal chain** for S denoted by $C(S)$ is the set of all events from the beginning of the chain to the first event involving an healthcare professional outside of S. Formally:

$$C(S) = \max_{q \in \{1,\dots,k\} : \{1,\dots,q\} \subseteq C^{-1}(S)} (c_1, \dots, c_q). \tag{2}$$

Example : lungs cancer

$$\underbrace{H \longrightarrow H \longrightarrow P}_{C(\{P,H\})} \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

The maximal chain

For each $S \subseteq N$, let $C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i)$,

Maximal chain

The **maximal chain** for S denoted by $C(S)$ is the set of all events from the beginning of the chain to the first event involving an healthcare professional outside of S. Formally:

$$C(S) = \max_{q \in \{1,\ldots,k\} : \{1,\ldots,q\} \subseteq C^{-1}(S)} (c_1, \ldots, c_q). \tag{2}$$

Example : lungs cancer

$$\underbrace{H \longrightarrow H \longrightarrow P}_{C(\{P,H\})} \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

The maximal chain

For each $S \subseteq N$, let $C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i)$,

Maximal chain

The **maximal chain** for S denoted by $C(S)$ is the set of all events from the beginning of the chain to the first event involving an healthcare professional outside of S. Formally:

$$C(S) = \max_{q \in \{1,\ldots,k\} : \{1,\ldots,q\} \subseteq C^{-1}(S)} (c_1, \ldots, c_q). \quad (2)$$

Example : lungs cancer

$$\underbrace{H \longrightarrow H \longrightarrow P}_{C(\{P,H\})} \longrightarrow S \longrightarrow H \longrightarrow H \longrightarrow P$$

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O'Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.

Approaches with Chain

There are two approaches taking into account the order of interventions:

	Chain
Optimistic vision	$w_B^C(S) = \min\left\{F; \sum_{c_q \in C(S)} p_{c_q}\right\}$
Pessimistic vision	$v_B^C(S) = \max\left\{0; F - \sum_{c_q \in C \setminus C(S)} p_{c_q}\right\}$

Approaches with Chain

There are two approaches taking into account the order of interventions:

	Chain
Optimistic vision	$w_B^C(S) = \min\left\{F; \sum_{c_q \in C(S)} p_{c_q}\right\}$
Pessimistic vision	$v_B^C(S) = \max\left\{0; F - \sum_{c_q \in C \setminus C(S)} p_{c_q}\right\}$

Approaches without chain

There are two approaches taking into account the total turnover of healthcare professionnals:

	Not chain
Optimistic vision	$u_B(S) = \min \left\{ F; \sum_{i \in S} p_i \lvert C^{-1}(i) \rvert \right\}$
Pessimistic vision	$z_B(S) = \max \left\{ 0; F - \sum_{i \in N \setminus S} p_i \lvert C^{-1}(i) \rvert \right\}$

Approaches without chain

There are two approaches taking into account the total turnover of healthcare professionnals:

	Not chain
Optimistic vision	$u_B(S) = \min\left\{F; \sum_{i\in S} p_i \lvert C^{-1}(i)\rvert\right\}$
Pessimistic vision	$z_B(S) = \max\left\{0; F - \sum_{i\in N\setminus S} p_i \lvert C^{-1}(i)\rvert\right\}$

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
  - A proportional allocation rule

Properties

Proposition 1

For any integrated healthcare problem B, games z_B, v_B^C, and w_B^C are convex.

- z_B is a classic bankruptcy game and is the dual of u_B.
- v_B^C and w_B^C are both convex (adapting to our richer framework the demonstration in Curiel et al. (1987)) and are not connected by duality relation.

Properties

Proposition 1

For any integrated healthcare problem B, games z_B, v_B^C, and w_B^C are convex.

- z_B is a classic bankruptcy game and is the dual of u_B.
- v_B^C and w_B^C are both convex (adapting to our richer framework the demonstration in Curiel et al. (1987)) and are not connected by duality relation.

Properties

Proposition 1

For any integrated healthcare problem B, games z_B, v_B^C, and w_B^C are convex.

- z_B is a classic bankruptcy game and is the dual of u_B.
- v_B^C and w_B^C are both convex (adapting to our richer framework the demonstration in Curiel et al. (1987)) and are not connected by duality relation.

The Shapley value

Desirability: For an arbitrary allocation f, for each $v \in G$, for each pair of distinct players $i, j \in N$, such that for each $S \subseteq N \backslash \{i, j\}$, $v(S \cup \{i\}) \geq v(S \cup \{j\})$, then $f_i(v) \geq f_j(v)$.

$q(i)$ is the index of the first intervention of the healthcare professional i

Proposition 2

The payoffs provided by the Shapley value of game v_B^C are ordered by the position of the first event involving each healthcare professional:

$$q(i) < q(j) \Longrightarrow Sh_i(v_B^C) \geq Sh_j(v_B^C),$$

because healthcare professional i is at least as desirable as healthcare professional j.

The Shapley value

Desirability: For an arbitrary allocation f, for each $v \in G$, for each pair of distinct players $i, j \in N$, such that for each $S \subseteq N \backslash \{i, j\}$, $v(S \cup \{i\}) \geq v(S \cup \{j\})$, then $f_i(v) \geq f_j(v)$.

$q(i)$ is the index of the first intervention of the healthcare professional i

Proposition 2

The payoffs provided by the Shapley value of game v_B^C are ordered by the position of the first event involving each healthcare professional:

$$q(i) < q(j) \Longrightarrow Sh_i(v_B^C) \geq Sh_j(v_B^C),$$

because healthcare professional i is at least as desirable as healthcare professional j.

The Shapley value

Proposition 3

The payoffs provided by the Shapley value of games u_B and z_B are ordered by the amount of turnover involving each healthcare professionals. For each $j \in N \backslash \{i\}$:

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Longrightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B) \\ Sh_i(z_B) \geq Sh_j(z_B) \end{cases}$$

because healthcare professional i is at least as desirable as healthcare professional j.

Proposition 4

Let $q^* = \max_{j \in N} q(j)$. Assume that $\sum_{q \geq q^*} p_{c_q} > F$, then the Shapley value of v_B^C provides equal payoffs to all healthcare professionals.

The Shapley value

Proposition 3

The payoffs provided by the Shapley value of games u_B and z_B are ordered by the amount of turnover involving each healthcare professionals. For each $j \in N \backslash \{i\}$:

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Longrightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B) \\ Sh_i(z_B) \geq Sh_j(z_B) \end{cases}$$

because healthcare professional i is at least as desirable as healthcare professional j.

Proposition 4

Let $q^* = \max_{j \in N} q(j)$. Assume that $\sum_{q \geq q^*} p_{c_q} > F$, then the Shapley value of v_B^C provides equal payoffs to all healthcare professionals.

The Shapley value

Proposition 3

The payoffs provided by the Shapley value of games u_B and z_B are ordered by the amount of turnover involving each healthcare professionals. For each $j \in N\backslash\{i\}$:

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Longrightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B) \\ Sh_i(z_B) \geq Sh_j(z_B) \end{cases}$$

because healthcare professional i is at least as desirable as healthcare professional j.

Proposition 4

Let $q^* = \max_{j \in N} q(j)$. Assume that $\sum_{q \geq q^*} p_{c_q} > F$, then the Shapley value of v_B^C provides equal payoffs to all healthcare professionals.

Priority rule

The priority rule (Moulin, 2000) is the allocation rule x^P which rewards the healthcare professionals in the order of their interventions until the fee F is depleted.

$\hat{q}$ is the penultimate event which is refunded and $\hat{q}_{+1}$ is the last (partially) refunded event:

$$\hat{q} = \text{argmax}\left\{q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{c_r} < F\right\}.$$

The set of all healthcare professionals who act before the depletion of F is:

$$\hat{S} = \left\{i \in N : q(i) \leq \hat{q}_{+1}\right\}.$$

Priority rule

The priority rule (Moulin, 2000) is the allocation rule x^P which rewards the healthcare professionals in the order of their interventions until the fee F is depleted.

$\hat{q}$ is the penultimate event which is refunded and $\hat{q}_{+1}$ is the last (partially) refunded event:

$$\hat{q} = \text{argmax}\Big\{q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{c_r} < F\Big\}.$$

The set of all healthcare professionals who act before the depletion of F is:

$$\hat{S} = \Big\{i \in N : q(i) \leq \hat{q}_{+1}\Big\}.$$

Priority rule

The priority rule (Moulin, 2000) is the allocation rule x^P which rewards the healthcare professionals in the order of their interventions until the fee F is depleted.

$\hat{q}$ is the penultimate event which is refunded and $\hat{q}_{+1}$ is the last (partially) refunded event:

$$\hat{q} = \operatorname{argmax}\left\{q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{c_r} < F\right\}.$$

The set of all healthcare professionals who act before the depletion of F is:

$$\hat{S} = \left\{i \in N : q(i) \leq \hat{q}_{+1}\right\}.$$

Priority rule

The priority rule for i is:

$$x_i^P(B) = \begin{cases} \sum_{q \leq \hat{q}: c_q = i} p_{c_q} & \text{if } c_{\hat{q}+1} \neq i, \\ \sum_{q \leq \hat{q}: c_q = i} p_{c_q} + F - \sum_{q=1}^{\hat{q}} p_{c_q} & \text{if } c_{\hat{q}+1} = i. \end{cases}$$

Proposition 5

The payoffs provided by the priority rule x^P in problem B are in the core of games v_B^C and w_B^C.

Priority rule

The priority rule for i is:

$$x_i^P(B) = \begin{cases} \sum_{q \leq \hat{q} : c_q = i} p_{c_q} & \text{if } c_{\hat{q}+1} \neq i, \\ \sum_{q \leq \hat{q} : c_q = i} p_{c_q} + F - \sum_{q=1}^{\hat{q}} p_{c_q} & \text{if } c_{\hat{q}+1} = i. \end{cases}$$

Proposition 5

The payoffs provided by the priority rule x^P in problem B are in the core of games v_B^C and w_B^C.

Proportional allocation rule

The proportional allocation rule y^P is the allocation rule which refunds the healthcare professionals in proportion to their turnover:

$$y_i^P(S) = \frac{\sum_{i \in S} p_i |C^{-1}(i)|}{\sum_{j \in N} p_j |C^{-1}(j)|} \times F.$$

Proposition 6

The payoffs provided by the proportional allocation rule $y^P(S)$ in problem B are not in the core of games w_B^C and u_B.

Proportional allocation rule

The proportional allocation rule y^P is the allocation rule which refunds the healthcare professionals in proportion to their turnover:

$$y_i^P(S) = \frac{\sum_{i \in S} p_i |C^{-1}(i)|}{\sum_{j \in N} p_j |C^{-1}(j)|} \times F.$$

Proposition 6

The payoffs provided by the proportional allocation rule $y^P(S)$ in problem B are not in the core of games w_B^C and u_B.

To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (Sh for games v_B^C and w_B^C and x^P)
- To refund more healthcare professionals with the highest claims (Sh for games z_B and u_B and y^P)
- To refund equally healthcare professionals when the treatment is long (Sh for v_B^C)

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	+	+	−	+
x^P	+	+	−	?
y^P	?	−	−	?

The symbol " + " means that the allocation rule belongs to the core of the considered game, the symbol " − " has the converse meaning and the symbol "?" means that it remains to prove whether the allocation rule is core element or not.

To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (Sh for games v_B^C and w_B^C and x^P)
- To refund more healthcare professionals with the highest claims (Sh for games z_B and u_B and y^P)
- To refund equally healthcare professionals when the treatment is long (Sh for v_B^C)

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	$+$	$+$	$-$	$+$
x^P	$+$	$+$	$-$	?
y^P	?	$-$	$-$	?

The symbol " $+$ " means that the allocation rule belongs to the core of the considered game, the symbol " $-$ " has the converse meaning and the symbol "?" means that it remains to prove whether the allocation rule is core element or not.

To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (Sh for games v_B^C and w_B^C and x^P)
- To refund more healthcare professionals with the highest claims (Sh for games z_B and u_B and y^P)
- To refund equally healthcare professionals when the treatment is long (Sh for v_B^C)

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	$+$	$+$	$-$	$+$
x^P	$+$	$+$	$-$	?
y^P	?	$-$	$-$	?

The symbol " $+$ " means that the allocation rule belongs to the core of the considered game, the symbol " $-$ " has the converse meaning and the symbol "?" means that it remains to prove whether the allocation rule is core element or not.

To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (Sh for games v_B^C and w_B^C and x^P)
- To refund more healthcare professionals with the highest claims (Sh for games z_B and u_B and y^P)
- To refund equally healthcare professionals when the treatment is long (Sh for v_B^C)

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	$+$	$+$	$-$	$+$
x^P	$+$	$+$	$-$	?
y^P	?	$-$	$-$	?

The symbol " $+$ " means that the allocation rule belongs to the core of the considered game, the symbol " $-$ " has the converse meaning and the symbol "?" means that it remains to prove whether the allocation rule is core element or not.

To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (Sh for games v_B^C and w_B^C and x^P)
- To refund more healthcare professionals with the highest claims (Sh for games z_B and u_B and y^P)
- To refund equally healthcare professionals when the treatment is long (Sh for v_B^C)

	$C(v_B^C)$	$C(w_B^C)$	$C(u_B)$	$C(z_B)$
Sh	$+$	$+$	$-$	$+$
x^P	$+$	$+$	$-$	?
y^P	?	$-$	$-$	?

The symbol " $+$ " means that the allocation rule belongs to the core of the considered game, the symbol " $-$ " has the converse meaning and the symbol "?" means that it remains to prove whether the allocation rule is core element or not.