A cooperative game approach to integrated health care

Guillaume Sekli

Games and Optimization - Saint-Etienne

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Introduction

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The question of how taking in charge chronic patient become more and more relevant in health systems which are very competitive and fragmentated (Brekke et al., 2021).

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Heavy surgical procedures or acute medical care that requires short-stay care, outpatient care, or home care.
⇒ The patient pays all individual prices

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A set of care provided for a given health condition, during a given period of time and by all the health professionals involved in the care. The care or health pathway includes prevention or health education activities, coordination, and patient support for care.
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Many experimentations in a lot of countries with good results (HCAAM, 2015; Busse and Stahl, 2014; Struijs and Baan, 2011; de Bakker et al., 2012).

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Bundled payment in integrated healthcare are surveyed by Rocks et al. (2020).
A bundled payment $B$ is a quadruplet $(N, \{p_i\}_{i \in N}, C, F)$:

- $N = \{1, \ldots, n\}$ is the set of all healthcare professionals.
- $p_i > 0$, the price of service provided by $i$.
- $C = (c_1, \ldots, c_k)$ a chain modeling the recovery path where each service $q \in \{1, \ldots, k\}$, $c_q \in N$ is identified by the corresponding provider.
- $F > 0$, the fee such as:

$$\sum_{q \in \{1, \ldots, k\}} p_{c_q} > F \quad (1)$$

We have to find an allocation $x \in \mathbb{R}^N$ such as:

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Example : Lungs cancer

- \( N = \{ P, S, H \} \) with \( P \) the practitioner, \( S \) the specialist and \( H \) the hospital

- \( p_P = 25, p_S = 45, p_H = 110 \).

- \( C = (c_1, c_2, c_3, c_4, c_5, c_6, c_7) \) is represented by:
  \[ H \rightarrow H \rightarrow P \rightarrow S \rightarrow H \rightarrow H \rightarrow P \]

- The total cost the patient should have paid is:
  \[ 110 + 110 + 25 + 45 + 110 + 110 + 110 + 25 = 535 \]

- Suppose a fee \( F = 400 \).
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Bankruptcy games

The problem $B$ can be apprehended by means of cooperative game theory and can be inspired by bankruptcy games literature (O’Neill, 1982; Aumann and Maschler, 1985):

- The total estate to share among healthcare professionals is $F$
- The claimants are healthcare professionals
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There are two original features:
- The order of interventions during the process
- The possibility for an healthcare professional to act several times during the process

There are bankruptcy games which taking into account the position of players (Ansink and Weikard, 2012) but without the possibility to be in more than one position.
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The chain and the turnover

The healthcare professionals have the possibility to act one time or more than once. The set of all events for a player \( i \in N \) is a correspondance \( N \rightarrow \{1, \ldots, k\} \) that associates to each \( i \in N \) one or more positions in the chain \( C \). This is done by the inverse function \( C^{-1}(i) \) defined as:

\[
C^{-1}(i) = \left\{ q \in \{1, \ldots, k\} : c_q = i \right\}, \quad \forall i \in N.
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The total turnover involving \( i \) is:

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\sum_{q \in C^{-1}(i)} p_{c_q} = |C^{-1}(i)| p_i.
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This turnover can be interpreted as the legitimate claim of health professional \( i \) or its bargaining power when sharing \( F \), which refers naturally to the bankruptcy approach.
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For each $S \subseteq N$, let $C^{-1}(S) = \bigcup_{i \in S} C^{-1}(i)$.

### Maximal chain

The **maximal chain** for $S$ denoted by $C(S)$ is the set of all events from the beginning of the chain to the first event involving an healthcare professional outside of $S$. Formally:

$$C(S) = \max_{q \in \{1, \ldots, k\} : \{1, \ldots, q\} \subseteq C^{-1}(S)} (c_1, \ldots, c_q).$$

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Different approaches

Four possibilities can be obtain by answering the two following questions:

- Shall we account for the position of the healthcare professionals within the recovery path?
- Should a coalition look at its opportunities with an optimistic or pessimistic view? (O’Neill, 1982; Aumann and Maschler, 1985)

We can split the approaches in two categories:

- Two games which take the chain into account.
- Two games which take the turnover into account.
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Results

There are two types of results:

- The study of the convexity property
- The application of three different allocation rules and the analyse of their belonging to the core of the games:
  - The Shapley value (Shapley, 1953)
  - The Priority rule (Moulin, 2000)
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Properties

Proposition 1

For any integrated healthcare problem $B$, games $z_B$, $v_B^C$, and $w_B^C$ are convex.

- $z_B$ is a classic bankruptcy game and is the dual of $u_B$.
- $v_B^C$ and $w_B^C$ are both convex (adapting to our richer framework the demonstration in Curiel et al. (1987)) and are not connected by duality relation.
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The Shapley value

**Desirability**: For an arbitrary allocation $f$, for each $v \in G$, for each pair of distinct players $i, j \in N$, such that for each $S \subseteq N \setminus \{i, j\}$, $v(S \cup \{i\}) \geq v(S \cup \{j\})$, then $f_i(v) \geq f_j(v)$.

$q(i)$ is the index of the first intervention of the healthcare professional $i$.

**Proposition 2**

The payoffs provided by the Shapley value of game $v^C_B$ are ordered by the position of the first event involving each healthcare professional:

$$ q(i) < q(j) \implies Sh_i(v^C_B) \geq Sh_j(v^C_B), $$

because healthcare professional $i$ is at least as desirable as healthcare professional $j$. 
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Proposition 3

The payoffs provided by the Shapley value of games $u_B$ and $z_B$ are ordered by the amount of turnover involving each healthcare professionals. For each $j \in N \setminus \{i\}$:

$$p_i|C^{-1}(i)| \geq p_j|C^{-1}(j)| \Rightarrow \begin{cases} Sh_i(u_B) \geq Sh_j(u_B) \\ Sh_i(z_B) \geq Sh_j(z_B) \end{cases}$$

because healthcare professional $i$ is at least as desirable as healthcare professional $j$.

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Let $q^* = \max_{j \in N} q(j)$. Assume that $\sum_{q \geq q^*} p_{cq} > F$, then the Shapley value of $v^{C}_B$ provides equal payoffs to all healthcare professionals.
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**Proposition 4**

Let $q^* = \max_{j \in \mathcal{N}} q(j)$. Assume that $\sum_{q \geq q^*} p_{cq} > F$, then the Shapley value of $v_B^C$ provides equal payoffs to all healthcare professionals.
Priority rule

The priority rule (Moulin, 2000) is the allocation rule $x^P$ which rewards the healthcare professionals in the order of their interventions until the fee $F$ is depleted.

$\hat{q}$ is the penultimate event which is refunded and $\hat{q} + 1$ is the last (partially) refunded event:

$$\hat{q} = \arg\max\left\{ q \in \{1, ..., k\} : \sum_{r=1}^{q} p_{cr} < F \right\}.$$  

The set of all healthcare professionals who act before the depletion of $F$ is:

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The priority rule for $i$ is:

\[
x^P_i(B) = \begin{cases} 
\sum_{q \leq \hat{q}: c_q = i} p_{c_q} & \text{if } c_{\hat{q}+1} \neq i, \\
\sum_{q \leq \hat{q}: c_q = i} p_{c_q} + F - \sum_{q=1}^{\hat{q}} p_{c_q} & \text{if } c_{\hat{q}+1} = i.
\end{cases}
\]

**Proposition 5**

The payoffs provided by the priority rule $x^P_i$ in problem $B$ are in the core of games $v^C_B$ and $w^C_B$. 

Guillaume Sekli

A cooperative game approach to integrated health care
The priority rule for $i$ is:

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**Proposition 5**

The payoffs provided by the priority rule $x^P$ in problem $B$ are in the core of games $v_C^B$ and $w_C^B$. 
The proportional allocation rule $y^P$ is the allocation rule which refunds the healthcare professionals in proportion to their turnover:

$$y^P_i(S) = \frac{\sum_{i \in S} p_i |C^{-1}(i)|}{\sum_{j \in N} p_j |C^{-1}(j)|} \times F.$$

**Proposition 6**

The payoffs provided by the proportional allocation rule $y^P(S)$ in problem $B$ are not in the core of games $w_B^C$ and $u_B$. 
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**Proposition 6**

The payoffs provided by the proportional allocation rule $y^P(S)$ in problem $B$ are not in the core of games $w^C_B$ and $u_B$. 
To summarize

Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (*Sh* for games $v_B^C$ and $w_B^C$ and $x^P$)
- To refund more healthcare professionals with the highest claims (*Sh* for games $z_B$ and $u_B$ and $y^P$)
- To refund equally healthcare professionals when the treatment is long (*Sh* for $v_B^C$)

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<tr>
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The symbol “+” means that the allocation rule belongs to the core of the considered game, the symbol “−” has the converse meaning and the symbol “?” means that it remains to prove whether the allocation rule is core element or not.
Three different allocation rules:

- To refund more healthcare professionals at the beginning of the process (\(Sh\) for games \(v^C_B\) and \(w^C_B\) and \(x^P\))
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